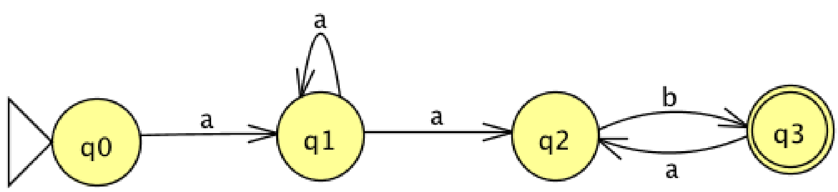
CSE105 Spring 2018 Practice Final

The final exam is on Saturday June 9, 8:00am-11:00am . The exam is cumulative and covers all the material from the quarter. In particular, you should study:

- Chapter 0: Sets, strings, languages, proofs.
- Chapter 1: Finite state machines, DFAs, NFAs, computation traces, regular languages, closure (of arbitrary sets under arbitrary operations, particularly in regards to regular languages), acceptance of a string by an NFA, subset construction, equivalence of DFAs and NFAs, regular expressions. Non regular languages, Pumping Lemma.
- Sections 2.1-2.3: Context-free grammars, context-free languages, derivations, ambiguous grammars, leftmost derivations, pushdown automata, non-context-free languages, closure of class of context-free languages under some set operations and not under others.
- Chapter 3: Turing machines, configurations, halting, computations, formal descriptions of TMs, implementation-level descriptions of TMs, high-level descriptions of TMs, TM variants (multiple tapes, non-determinism, enumerators), equivalence of TM variants, Church-Turing thesis.
- Chapter 4: Turing-recognizable languages, Turing-decidable languages, closure properties, decidable problems, encoding of objects as strings, countable and uncountable sets, diagonalization, decision problems about languages (A_{TM}, E_{TM}, EQ_{TM}, A_{DFA},E_{DFA}, EQ_{DFA}, A_{CFG}, E_{CFG}, etc.)
- Chapter 5: Computable functions, Mapping reductions, undecidable languages, unrecognizable languages.
- (Briefly) Chapter 7: Time complexity, P, NP, NP-complete.
- All assigned homework questions, and solutions (available on Piazza).
- Midterm exams (available on Gradescope) and practice midterm exams (available on website).

There will be a review session on Wednesday, June 6 7:00pm-9:50pm in Galbraith 242 to answer questions about this practice exam. We will also review in lecture on Friday, June 8.

(1) Consider the following state diagram of an NFA, N, over the alphabet $\{a, b\}$.



(a) What is the formal definition of N? I.e., what is $(Q, \Sigma, \delta, q_0, F)$?
(b) List out all the possible computations of N on the string $aaba$. Do any of these computations get stuck? Is $aaba \in L(N)$?
(c) Draw the state diagram of a DFA that recognizes the same language as this NFA.
(d) Write a regular expressions that describes the same language as this NFA.

(2) Show that the class of regular languages over the alphabet $\{a, b, c\}$ is closed under the operation $ForceB(L)$, defined as

$$ForceB(L) = \{forceb(w) \mid w \in L\}$$

where $forceb$ is an operation on a string that replaces the first character of the string with a b. More precisely, $forceb(\varepsilon) = \varepsilon$ and $forceb(xy) = by$ for any $x \in \{a, b, c\}, y \in \{a, b, c\}^*$.

(3) Is each of the following languages over $\{0, 1\}$ regular? Prove your answer.
(a) $\{0^m1^n \mid m \neq n\}$
(b) $\{0^m1^n \mid m, n \geq 3\}$
(c) $\{0^m1^n \mid m = n + 3\}$

(4) Consider the language

$$\{w \in L(a^*b^*a^*) \mid \text{the number of } a\text{'s in } w \text{ equals the number of } b\text{'s in } w\}.$$

(a) Build a CFG that generates this language.
(b) Give an implementation-level description of a PDA recognizing this language.
(c) Give the state diagram of the PDA you described in part (b).
(d) Is this language regular? Prove your answer.

(5) Prove using PDAs that if C is a context-free language and R is a regular language, then $C \cap R$ is context-free. In other words, the set of context-free languages is closed under the operation of intersection with a regular language.

(6) Consider the language

$$L = \{w \in \{a, b\}^* \mid |w| > 0\}$$

(a) Give the formal definition of a Turing machine deciding L.

(b) Give an implementation-level description of a Turing machine that recognizes L but is not a decider.

(7) Let

$$BAL_{DFA} = \{\langle M \rangle \mid M \text{ is a DFA that accepts some binary string containing an equal number of 0s and 1s}\}.$$

Prove that BAL_{DFA} is decidable.

(8) Let $EVEN_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine, and } |w| \text{ is even for all strings } w \in L(M)\}$. Prove that $EVEN_{TM}$ is undecidable.

(9) Consider the following two Turing machines over the alphabet Σ, defined by the high-level descriptions

$M_1 =$ "On input x, where x is a string over Σ,
1. Halt and accept."

$M_2 =$ "On input $\langle M \rangle$, where M is a Turing machine,
1. Halt and reject."

For each of the following, choose Yes, No, or Type Error.

(a) Is $\langle M_1 \rangle \in L(M_2)$?
(b) Is $\langle M_2 \rangle \in L(M_1)$?
(c) If $D = \{\langle M \rangle \mid \langle M \rangle \notin L(M)\}$, is $\langle M_1 \rangle \in D$?
(d) If $D = \{\langle M \rangle \mid \langle M \rangle \notin L(M)\}$, is $\langle M_2 \rangle \in D$?
(e) If $D = \{\langle M \rangle \mid \langle M \rangle \notin L(M)\}$, is $\langle D \rangle \in D$?
(f) Is $L(M_1)$ mapping reducible to $L(M_2)$?
(g) Is $L(M_2)$ mapping reducible to $L(M_1)$?

(10) True/ False

(a) A DFA may reject a string without reading all of it.
(b) An NFA may reject a string without reading all of it.
(c) A PDA may accept a string without reading all of it.
(d) A TM may accept a string without reading all of it (i.e. without scanning each of its symbols).
(e) For any CFG G, if a string is in $L(G)$ then it is derived by exactly one leftmost derivation in G.
(f) There are countably many different subsets of $\{1\}^*$.
(g) The Halting Problem is the problem of deciding whether a given TM M accepts a given string w.
(h) $DTIME(n) \subseteq DTIME(n^2)$
(i) $DTIME(n^5) \subseteq NTIME(n^5)$
(j) Each language in NP cannot be decided by any TM in polynomial time.