CSE 105
THEORY OF COMPUTATION

Spring 2018

http://cseweb.ucsd.edu/classes/sp18/cse105-ab/

Today's learning goals

Sipser Section 1.1

- Prove closure properties of the class of regular languages
- Apply closure properties to conclude that a language is or isn't regular

Regular languages

Sipser p. 35 Def 1.5

- DFA M over the alphabet Σ
  - For each string w over Σ, M either accepts w or rejects w
  - The **language recognized by M** is the set of strings M accepts

  a.k.a. the **language of M** is the set of strings M accepts

  a.k.a. **L(M)** = { w | w is a string over Σ and M accepts w}

A language is **regular** iff there is some finite automaton that recognizes exactly it.

Justification?

To prove that the DFA we build, M, actually recognizes the language L

WTS L(M) = L

(1) Is every string accepted by M in L?

(2) Is every string from L accepted by M?

or contrapositive version: Is every string rejected by M not in L?

A useful (optional) bit of terminology

When is a string accepted by a DFA?

Computation of M on w: *where do we land when start at q_0 and read each symbol of w one-at-a time?*

$M = (Q, \Sigma, \delta, q0, F)$

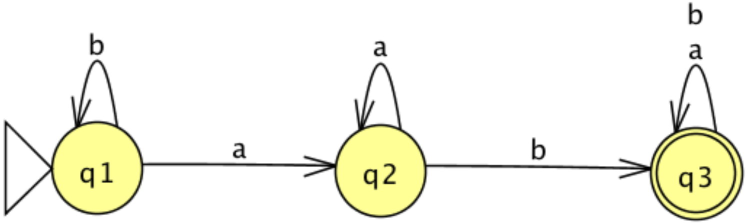
$\delta^*((q, w)) =$

Recursively defined function

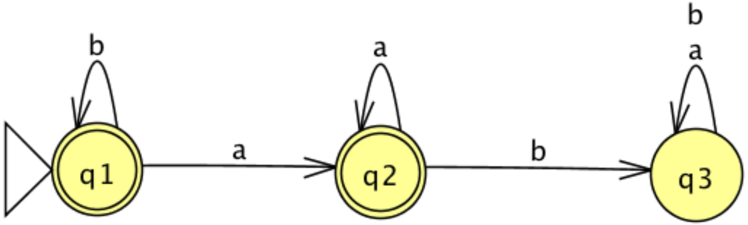
Building DFA

Formally,

{w | w contains the substring ab}



{w | w doesn't contain the substring ab}



Complementation

Claim: If A is a regular language over Σ, then so is $\overline{A}$

aka "the class of regular languages is closed under complementation"

Complementation

Claim: If A is a regular language over Σ, then so is $\overline{A}$

aka "the class of regular languages is closed under complementation"

Proof: Let A be a regular language. Then there is a DFA M=(Q,Σ,δ,q0,F) such that L(M) = A. We want to build a DFA whose language is $\overline{A}$. Define

M' = ?

Claim of Correctness L(M') = $\overline{A}$

Proof of claim…

Why closure proofs?

- General technique of proving a new language is regular
- Stretch the power of the model
- Puzzle!

Set operations

Input language (s) → OPERATION → Output language

The class of regular languages is closed under …

Complementation ✔

Kleene star ?

Concatenation ?

Union ?

Intersection ?

Set difference ?

Union

Sipser Theorem 1.25 p. 45

Theorem: The class of regular languages over fixed alphabet Σ is closed under the **union operation**.

Proof:

What are we proving here?

A. For any set A, if A is regular then so is A U A.
B. For any sets A and B, if A U B is regular, then so is A.
C. For two DFAs M1 and M2, M1 U M2 is regular.
D. None of the above.
E. I don't know.

Union

Sipser Theorem 1.25 p. 45

Theorem: The class of regular languages over fixed alphabet Σ is closed under the union operation.

Proof: Let A1, A2 be any two regular languages over Σ.

WTS that A1 U A2 is regular.

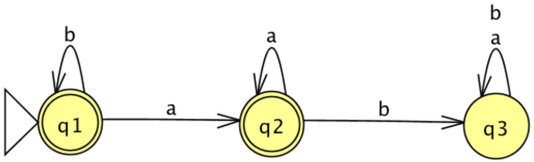
Goal: build a machine that recognizes A1 U A2.

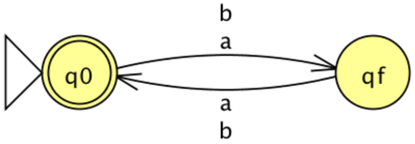
Union

Sipser Theorem 1.25 p. 45

Goal: build a machine that recognizes A1 U A2.

Strategy: use machines that recognize each of A1, A2.


b
a
b
a
b
a
q1
q2
q3


b
a
a
b
q0
qf

"Run in parallel"

Building M = (Q1xQ2, Σ, δ, ?, ?)

Given M1 = (Q1, Σ, δ1, q1, F1) M2 = (Q2, Σ, δ2, q2, F2)

Start state:

Accept state(s):

Transition function:

"Run in parallel"

Building $M = (Q1 \times Q2, \Sigma, \delta, ?, ?)$

Given $M1 = (Q1, \Sigma, \delta 1, q1, F1)$ $M2 = (Q2, \Sigma, \delta 2, q2, F2)$

Start state:

Accept state(s):

Transition function:

The set of accepting states for M is

A. F1 x F2
B. { (r,s) | r is in F1 and s is in F2 }
C. { (r,s) | r is in F1 or s is in F2 }
D. F1 U F2
E. I don't know.

"Run in parallel"

Building M = (Q1xQ2, Σ, δ, ?, ?)

Given M1 = (Q1, Σ, δ1, q1, F1) M2 = (Q2, Σ, δ2, q2, F2)

Start state:

Accept state(s):

Transition function:

Union

Sipser Theorem 1.25 p. 45

Proof: Let A1, A2 be any two regular languages over Σ.
Given M1 = (Q1,Σ,δ1,q1,F1) such that L(M1) = A1 and
M2 = (Q2,Σ,δ2,q2,F2) such that L(M2) = A2.

WTS that A1 U A2 is regular.

Define
M = (Q1xQ2,Σ,δ,(q1,q2),{(r,s) in Q1xQ2 | r in F1 or s in F2})
with δ(((r,s), x)) = (δ1(r,x), δ2(s,x)) for (r,s) in Q1xQ2, x in Σ.
Why does L(M) = A1 U A2 ?

Aside: Intersection

- *How would you prove that the class of regular languages is closed under intersection?*
- *Can you think of **more than one** proof strategy?*

$$A \cap B = \{ x \mid x \text{ in } A \text{ and } x \text{ in } B \}$$

General proof structure/strategy

Theorem: For any L over Σ, if L is regular then [the result of some operation on L] **is also regular.**

Proof:

Given name variables for sets, machines assumed to exist.

WTS state goal and outline plan.

Construction using objects previously defined + new tools working towards goal. Give formal definition and explain.

Correctness prove that construction works.

Conclusion recap what you've proved.

Before/For next time

- Work on Individual Homework 1 **due Tuesday**
- Work on Group Homework 1 **due Saturday**

Pre class-reading for Wednesday:

- Page 48 (Figure 1.27 and description below it)
- Example 1.35 on page 52

Payoff

{ w | w contains neither the substrings aba nor baab}

Is this a regular set?

Payoff

{ w | w contains neither the substrings aba nor baab}

Is this a regular set?

A = { w | w contains aba as a substring}

B = { w | w contains baab as a substring}

$$\bar{A} \cap \bar{B} = \overline{A \cup B}$$