CSE 105
THEORY OF COMPUTATION

Spring 2018

http://cseweb.ucsd.edu/classes/sp18/cse105-ab/

Today's learning goals

Sipser Section 1.1

- Design an automaton that recognizes a given language.
- Specify each of the components in a formal definition of an automaton.
- Prove that an automaton recognizes a specific language.

Deterministic finite automaton

Sipser p. 35 Def 1.5

A **finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

1. Q is a finite set called the states
2. Σ is a finite set called the alphabet
3. $\delta : Q \times \Sigma \to Q$ is the transition function
4. $q_0 \in Q$ is the start state
5. $F \subseteq Q$ is the set of accept states.

Can there be more than one **start state** in a finite automaton?

A. Yes, because of line 4.
B. No, because of line 4.
C. I don't know

Deterministic finite automaton

Sipser p. 35 Def 1.5

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How many outgoing arrows from each state?

A. May be different number at each state.
B. Must be 2.
C. Must be |Q|.
D. Must be |Σ|
E. I don't know.

An example

Define M = $(\{q1, q2, q3, q4\}, \{a, b\}, \delta, q1, \{q4\})$ where the function δ is specified by its table of values:

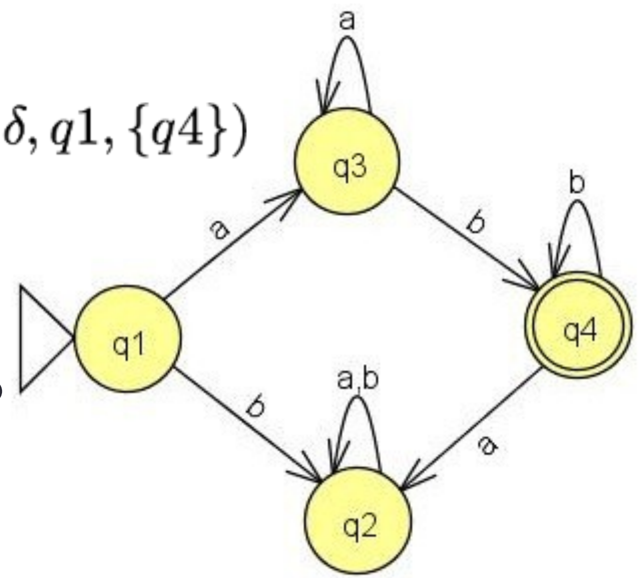
Input in Q x Σ	Output in Q
(q1,a)	q3
(q2,a)	q2
(q3,a)	q3
(q4,a)	q2

Input in Q x Σ	Output in Q
(q1,b)	q2
(q2,b)	q2
(q3,b)	q4
(q4,b)	q4

Draw the state diagram for the DFA with this formal definition.

An example

$(\{q1, q2, q3, q4\}, \{a, b\}, \delta, q1, \{q4\})$



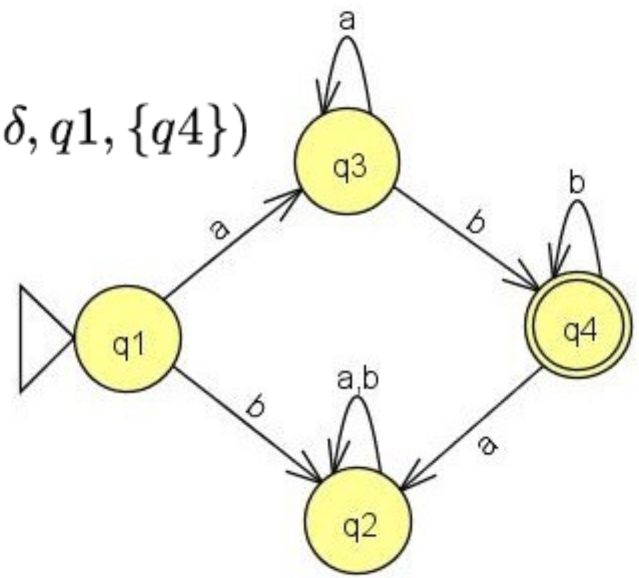
What's an example of a

- length 1 string accepted by this DFA?
- length 1 string rejected by this DFA?

- length 2 string accepted by this DFA?
- length 2 string rejected by this DFA?

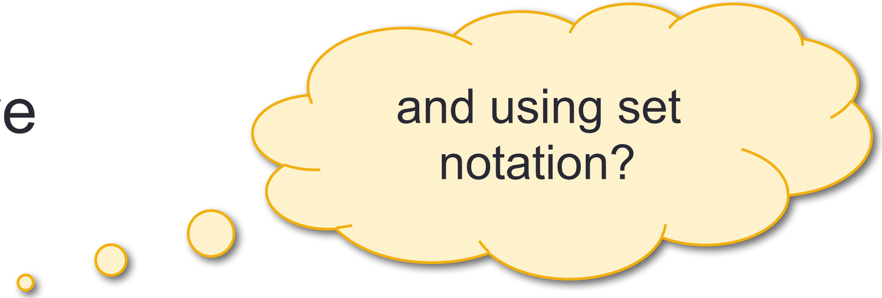
An example

$(\{q1, q2, q3, q4\}, \{a, b\}, \delta, q1, \{q4\})$



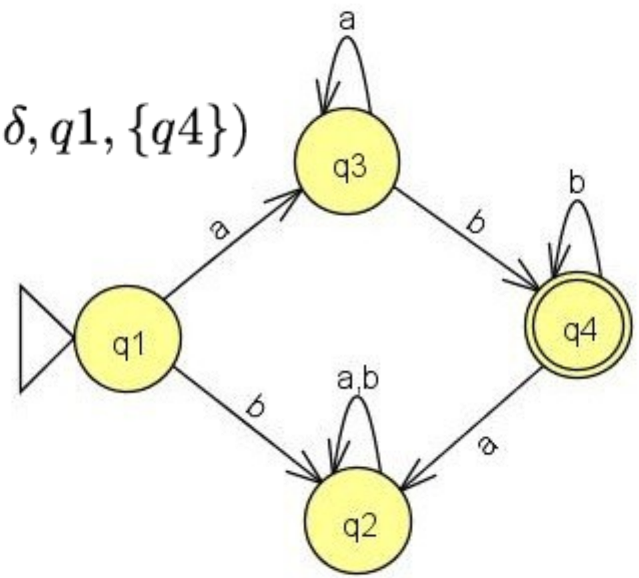
What's the best description of the language recognized by this DFA?

A. Starts with b and ends with a or b
B. Starts with a and ends with a or b
C. a's followed by b's
D. More than one of the above
E. I don't know.



An example

$(\{q1, q2, q3, q4\}, \{a, b\}, \delta, q1, \{q4\})$

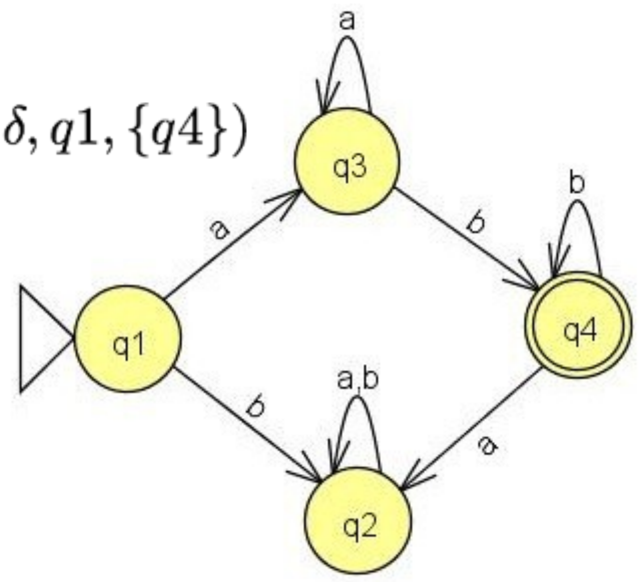


This DFA recognizes
the language of all strings
of the form a's followed by b's

i.e. $\{ a^n b^k \mid n,k \geq 1\}$

An example

$(\{q1, q2, q3, q4\}, \{a, b\}, \delta, q1, \{q4\})$



{ $a^n b^k$ | n,k ≥ 1}

Is this the same as the language described by

A. a*b*
B. a(ba)*b
C. a* U b*
D. (aaa)*
E. None of the above

Regular languages

Sipser p. 35 Def 1.5

- DFA M over the alphabet Σ
  - For each string w over Σ, M either accepts w or rejects w
  - The **language recognized by M** is the set of strings M accepts, and is also known as the **language of M** and written **L(M)**

A language is **regular** iff there is some finite automaton that recognizes exactly it.

Building DFA

Definition: A language is **regular** means there is some DFA that recognizes it.

Typical questions

Define a DFA which recognizes the language L.

or

Prove that the (given) language L is regular.

Building DFA

Example

Define a DFA which recognizes

{ w | w has at least 2 a's}

Bonus: what would you change if "at most" instead?

Justification?

To prove that the DFA we build, M, actually recognizes the language L

WTS L(M) = L

(1) Is every string accepted by M in L?

(2) Is every string from L accepted by M?

or contrapositive version: Is every string rejected by M not in L.

Building DFA

Remember

States are our only (computer) memory.

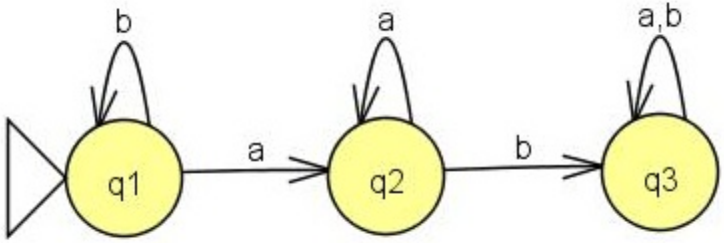
Design and pick states with specific roles / tasks in mind.

"Have not seen any of desired pattern yet"

"Trap state"

Specifying an automaton

({q1,q2,q3}, {a,b}, δ, q1, ?)

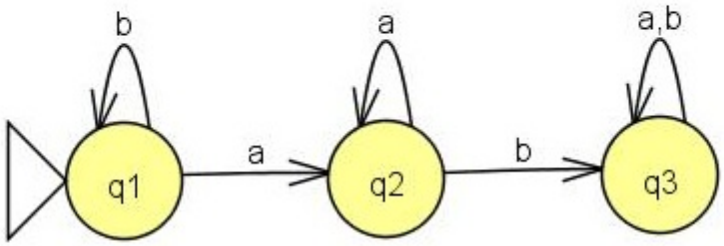


What state(s) should be in F so that the language of this machine is
{ w | ab is a substring of w}?

A. {q2}
B. {q3}
C. {q1,q2}
D. {q1,q3}
E. I don't know.

Specifying an automaton

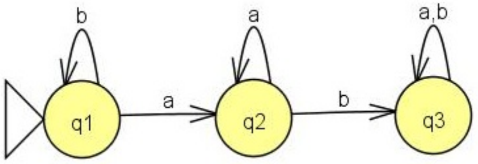
({q1,q2,q3}, {a,b}, δ, q1, ?)



What state(s) should be in F so that the language of this machine is
{ w | b's never occur after a's in w}?

A. {q2}
B. {q3}
C. {q1,q2}
D. {q1,q3}
E. I don't know.

Building DFA



Formally,

{w | w contains the substring ab}

{w | w doesn't contain the substring ab}

For next time

- Finish Individual Homework 0 **due Saturday**
- Review quiz 1 **due Sunday** (for credit)
- Read Individual Homework 1 **due Tuesday**

Pre class-reading for Monday:

Theorem 1.25, Theorem 1.26

Complementation

Claim: If A is a regular language over $\{0,1\}^*$, then so is $\overline{A}$

aka "the class of regular languages is closed under complementation"