CSE 105
THEORY OF COMPUTATION

Spring 2018

http://cseweb.ucsd.edu/classes/sp18/cse105-ab/

Today's learning goals

Sipser Ch 4.2

- Trace high-level descriptions of algorithms for computational problems.
- Use counting arguments to prove the existence of unrecognizable (undecidable) languages.
- Use diagonalization in a proof of undecidability.

Computational problems

A computational problem is **decidable** iff the language encoding the problem instances is decidable.

In Sipser 4.1: The computational problems below

A_{DFA}, A_{NFA}, A_{REX}, A_{CFG}

E_{DFA}, E_{NFA}, E_{REX}, E_{CFG}

EQ_{DFA}, EQ_{NFA}, EQ_{REX}

are all decidable

E_{REX} decidable

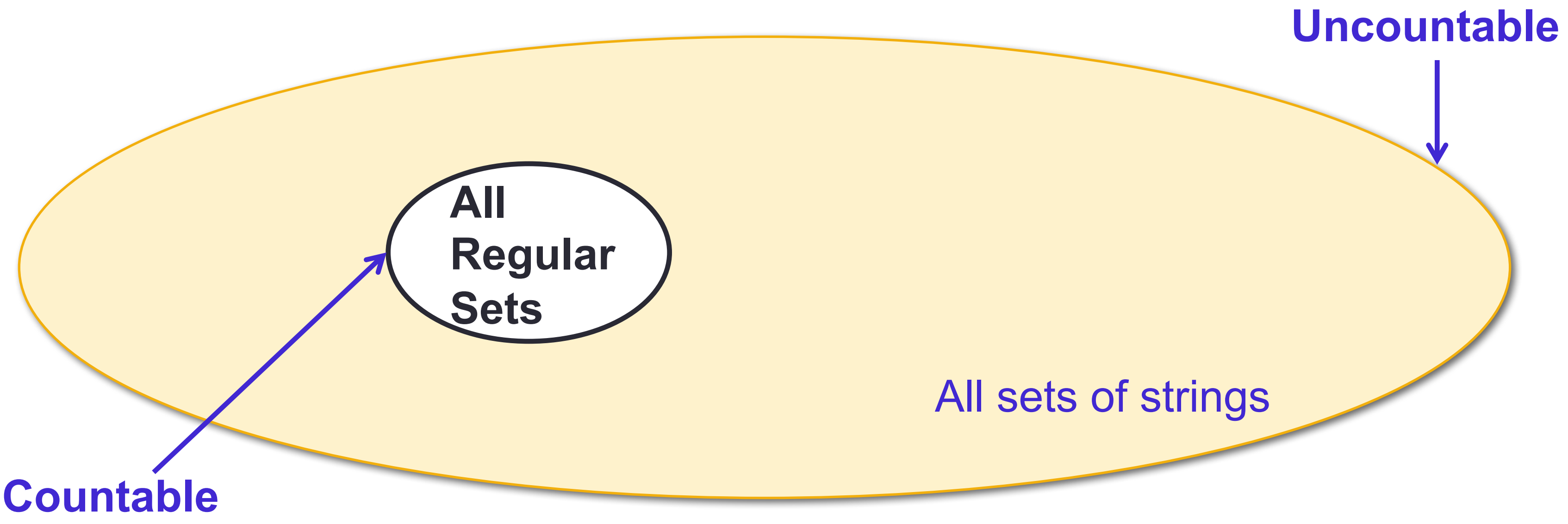
Undecidable?

- There are many ways to prove that a problem ***is*** decidable.
- How do we find (and prove) that a problem ***is not*** decidable?

Counting arguments

Before we proved the Pumping Lemma …

We proved there was a set that was not regular because



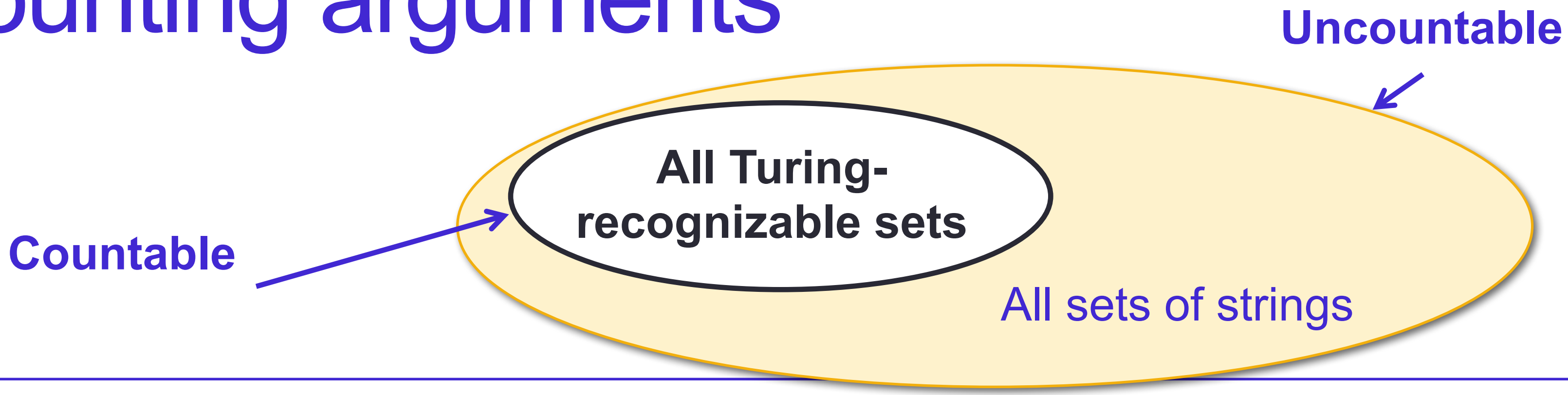
Reminder: countable/ uncountable

Sets A and B have the **same size** $|A| = |B|$ means there is a function between them that is both one-to-one and onto.

Countable (finite or same size as N)	Uncountable
N, Z, Q	R
Σ^*	{ infinite sequences over Σ}
The set of all TMs	$P(\Sigma^*)$

Sipser p. 202-204

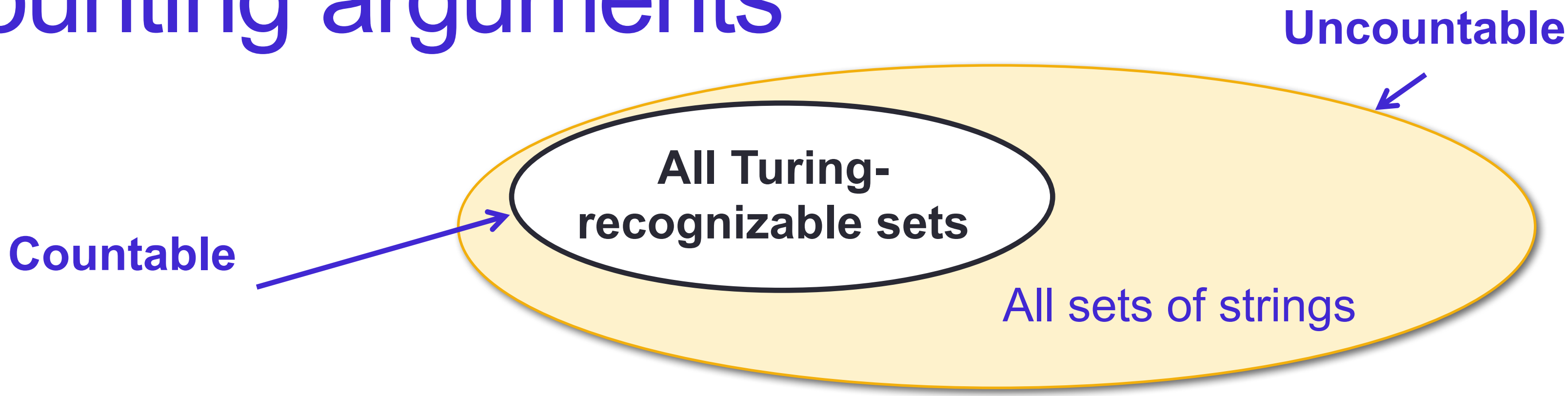
Counting arguments



Why is the set of Turing-recognizable languages **countable**?

A. It's equal to the set of all TMs, which we showed is countable.
B. It's a subset of the set of all TMs, which we showed is countable.
C. Each Turing-recognizable language is associated with a TM, so there can be no more Turing-recognizable languages than TMs.
D. More than one of the above.
E. I don't know.

Counting arguments



Is the set of Turing-decidable sets countable?

Satisfied?

- Maybe not …
- What's a specific example of a language that is not Turing-recognizable? or not Turing-decidable?
- *Idea: consider a set that, were it to be Turing-decidable, would have to "talk" about itself, and contradict itself!*

A_{TM}

Recall $A_{DFA} = \{<B,w> \mid B \text{ is a DFA and } w \text{ is in } L(B) \}$

$A_{TM} = \{<M,w> \mid M \text{ is a TM and } w \text{ is in } L(M) \}$

What is A_{TM} ?

A. A Turing machine whose input is codes of TMs and strings.

B. A set of pairs of TMs and strings.

C. A set of strings that encode TMs and strings.

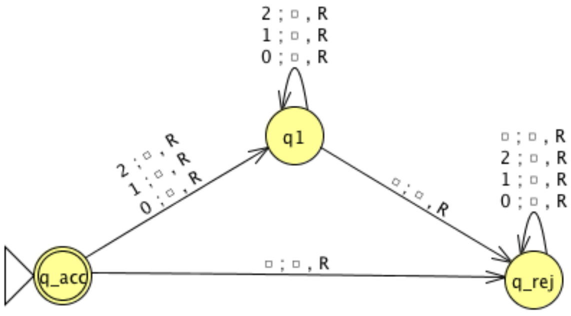
D. Not well defined.

E. I don't know.

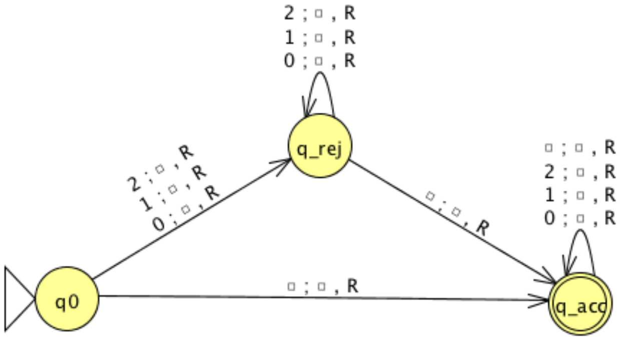
A_{TM} = {<M,w> | M is a TM and w is in L(M) }

From HW5

M_1



M_2



Which of the following are in A_{TM}?

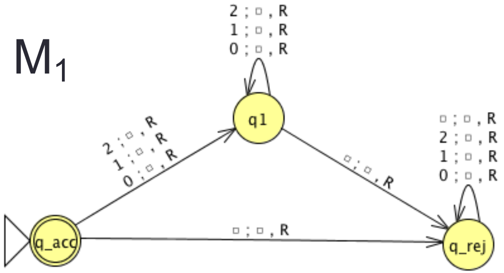
<M_1> <M_2> <M_1, ε> <M_2, ε > < M_1,<M_2>> < M_2,<M_1>>

A_{TM} = {<M,w> | M is a TM and w is in L(M) }

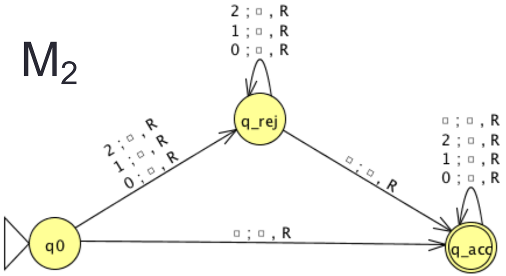
Define the TM N = "On input <M,w>:

1. Simulate M on w.
2. If M accepts, accept. If M rejects, reject."

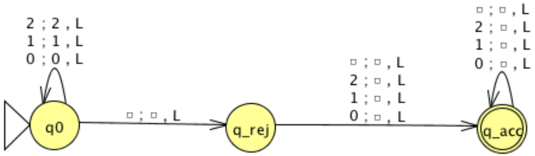
M_1



M_2



M_4



A. N rejects <M_1, 0>

B. N accepts <M_2>

C. N rejects <M_4, 1>

D. N recognizes A_{TM}

E. More than one of the above.

A_{TM}

$A_{DFA} = \{\langle B,w\rangle \mid B \text{ is a DFA and } w \text{ is in } L(B)\}$

Decider for this set simulates arbitrary DFA

$A_{TM} = \{\langle M,w\rangle \mid M \text{ is a TM and } w \text{ is in } L(M)\}$

Diagonalization proof: A_{TM} not decidable *Sipser p. 207*

Assume, towards a contradiction, that it is.

Call $\mathbf{M_{ATM}}$ the decider for A_{TM}:

$\mathbf{M_{ATM} \neq N}$

For every TM M and every string w,

- Computation of M_{ATM} on <M,w> halts and accepts if w is in L(M).
- Computation of M_{ATM} on <M,w> halts and rejects if w is not in L(M).

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, towards a contradiction, that M_{ATM} decides A_{TM}

Define the TM D = "On input <M>:

1. Run M_{ATM} on <M, <M>>.
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, towards a contradiction, that M_{ATM} decides A_{TM}

Define the TM D = "On input <M>:

1. Run M_{ATM} on <M, <M>>.
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

Which of the following computations halt?

A. Computation of D on <M_1>
B. Computation of D on <M_4>
C. Computation of D on <D>
D. All of the above.
E. None of the above.

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, towards a contradiction, that M_{ATM} decides A_{TM}

Define the TM D = "On input <M>:

1. Run M_{ATM} on <M, <M>>.
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

Consider **running D on input <D>**. Because D is a decider:

- **either computation halts and accepts** …
- **or computation halts and rejects** …

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, t

Def

1. R
2. If

Diagonalization???

Self-reference

"Is <D> an element of L(D)?"

Consider **running** **on input <D>**. Because D is a decider:

- **either computation halts and accepts …**
- **or computation halts and rejects …**

Do we have to diagonalize?

- Next time (after exam): comparing difficulty of problems.

Next time: Exam 2

Bring ID, note card

Check seat maps before coming to class

A_{TM}

- Recognizable
- Not decidable

Fact (from discussion section): A language is decidable iff it and its complement are both recgonizable.

Corollary: The complement of A_{TM} is **unrecognizable**.