CSE 105
THEORY OF COMPUTATION

Spring 2018

http://cseweb.ucsd.edu/classes/sp18/cse105-ab/

Today's learning goals

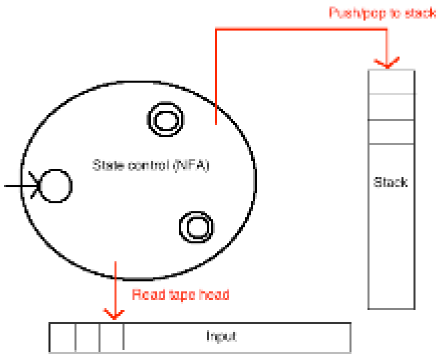
Sipser Section 2.1

- Identify the components of a formal definition of a CFG
- Derive strings in the language of a given CFG
- Determine the language of a given CFG
- Design a CFG generating a given language

Mid quarter feedback: 1 participation point
https://goo.gl/forms/eo14OwIc84Bzjvda2

PDA: NFA as ??: RegExp

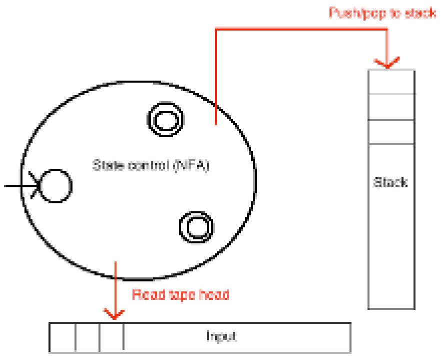
- Automata
  - String is read by the machine one character at a time, from left to right.
  - Determine if computation is successful by checking if entire string was read, and if land at an accept state.



Note: PDA based on **N**FA; can't always be determinized.

PDA: NFA as ??: RegExp

- Automata
  - String is read by the machine one character at a time, from left to right.
  - Determine if computation is successful by checking if entire string was read, and if land at an accept state.
- Regular expressions and ??
  - Derive all strings in the language by following rules for required patterns.



Context-free grammar

Informally, a collection of rules used to *create* string.

CFGs *generate* languages.

Some sample rules:

$$S \rightarrow \boxed{aTb}$$
$$T \rightarrow aT$$
$$T \rightarrow bTS$$
$$S \rightarrow \varepsilon$$

LHS RHS

More formally...

Context-free grammar

Sipser Def 2.2, page 102

(V, Σ, R, S)

Variables: finite set of (usually upper case) variables **V**

Terminals: finite set of alphabet symbols **Σ** $V \cap \Sigma = \emptyset$

Rules/Productions: finite set of allowed transformations **R**

$A \to u$ $A \in V, u \in (V \cup \Sigma)^*$

Start variable: origination of each derivation **S**

Derivation

Set of variables

Set of terminals

$G = (\{S\}, \{0\}, R, S)$

with the rules

$R = \{S \rightarrow 0S,\ S \rightarrow 0\}$

Sample derivation:

$S \Rightarrow 0S \Rightarrow 00S \Rightarrow 000$

Start variable

One-step application of rule

String of terminals

Context-free language

Sipser p. 104

The **language generated by a CFG** (V, Σ, R, S) is

{ w in Σ^* | starting with the Start variable and applying sequence of rules, can derive w on RHS}

If G = (V, Σ, R, S) the language generated by G is denoted L(G).

Notation:

$$S \implies^* w$$

Terminology: sequence of rule applications is **derivation**

Context-free language

Sipser p. 104

The language generated by CFG (V, Σ, R, S) is

{ w in Σ^* | starting with the Start variable and applying sequence of rules, can derive w on RHS}.

$L(G) = \boxed{}$

What is the language generated by the CFG ({S}, {0}, R, S) with the rules R = {S → 0S, S → 0} ?

start

~~A.~~ {0}

~~B.~~ {0, 0S}

C. {0, 00, 000, …}

~~D.~~ {ε, 0, 00, 000, …}

E. I don't know.

$000 \in L(G)$

$A \subsetneq L(G)$

Context-free language

Sipser p. 104

What is the language of the CFG ({S}, {0,1}, R, S) with rules

$$S \rightarrow 0S$$
$$S \rightarrow 1S$$
$$S \rightarrow \varepsilon$$

"or" "or"

$S \rightarrow 0S \mid 1S \mid \varepsilon$

1 2 3

A. L(0*1*)
B. L(0* U 1*)
C. L((0 U 1)*)
D. L ((0*1*)*)
E. I don't know.

$\underline{\varepsilon\ 1}\ \underline{0\ 1} = 101$

101

Designing a CFG

L = { abba }

$S \Rightarrow aT \Rightarrow abV \Rightarrow abbW \Rightarrow abba$

Which CFG generates L?

A. ({S, T, V, W} , {a, b}, {S → aT , T → bV , V → bW , W → a }, S)

B. ({Q}, {a,b}, { Q → abba } , Q)

$Q \Rightarrow abba$

C. ({X, Y} , {a,b}, { X → aYa , Y → bb } , X)

$X \Rightarrow aYa \Rightarrow abba$

D. All of the above

E. None of the above

Context-free languages

- L (00*)
- L((0U1)*)
- { abba }

Is any nonregular set context-free?

What about the languages that are recognized by PDAs?

Designing a CFG

We know this set is not regular!

$L = \{ a^n b^n \mid n \geq 0 \}$

$X \rightarrow aXb$

$X \rightarrow \varepsilon$

aabb?

$X \Rightarrow aXb \Rightarrow \ldots aaXbb \Rightarrow$

aabb

i.e. $(\{X\}, \{a, b\}, \{X \rightarrow aXb \mid \varepsilon\}, X)$

Designing a CFG

$$L = \{ a^n b^n \mid n \geq 0 \}$$

One approach:

- what is shortest string in the language?
- how do we go from shorter strings to longer ones?

Context-free languages

- L (00*)
- L((0U1)*)
- { abba }
- { $a^n b^n$ | n ≥ 0 }

$0^i \; 1^{j-i} 1^i$

Ex: { $0^i 1^j$ | j ≥ i ≥ 0 }   recognizable by a PDA

- add 0s & 1s at same time
- not too many 0s

$X \rightarrow 0X1 \mid X1 \mid \varepsilon \mid 1$

PDAs and CFGs are equally expressive

Theorem 2.20: A language is context-free if and only if some nondeterministic PDA recognizes it.

Consequences

- Quick proof that every regular language is context free
- To prove closure of class of CFLs under a given operation, can choose two modes of proof (via CFGs or PDAs) depending on which is easier