
INSTRUCTIONS

This **Individual HW8** covers the material from Chapter 7 you should be comfortable with. You can use it to help you study for the final exam.

READING Sipser Highlights from Chapter 7.

KEY CONCEPTS Running time of a Turing machine, time complexity classes, polynomial time (P), proving that a problem is in P , NP , proving that a problem is in NP , polynomial-time reduction, NP -completeness, examples of NP -complete problems, using reduction to prove NP -completeness

1.
 - a. Prove that P is closed under complementation.
 - b. It is not currently known whether NP is closed under complementation. Would knowing that (i) NP is closed under complementation or that (ii) NP is not closed under complementation give us any information about whether $P = NP$? Briefly justify your answer.
2. Let n be the number of states in a DFA. Show that $E_{DFA} \in P$ by showing that the algorithm in the proof of Theorem 4.4 (that E_{DFA} is a decidable language) runs in polynomial time (as a function of n).
3. In class we gave a polynomial-time reduction from 3SAT to CLIQUE. You can also find the details on page 302 of the textbook. Apply this reduction to the formula

$$\phi = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_2) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee x_3)$$

with 4 clauses. (We are using the book's convention that \bar{x} means $\neg x$). Is the formula satisfiable? Does the graph have a 4-clique? *Hint: the answers to these questions should both be yes or both be no.*