CSE 105
Group Homework 7
Due: Saturday June 5, 2018 at 11:00PM (on Gradescope)

INSTRUCTIONS
One member of the group should upload your group submission to Gradescope. During the submission process, they will be prompted to add the names of the rest of the group members. All group members' names and PIDs should be on **each** page of the submission.

Your homework must be typed. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning.

READING Sipser Sections 4.2, 5.1 (briefly), 5.3

KEY CONCEPTS Undecidable problems, recognizable and co-recognizable problems, unrecognizable problems, computable functions, mapping reductions.

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1. (10 points) Fix $\Sigma = \{0, 1\}$ for this question. **True/False** Briefly justify each answer:

a. Every language mapping reduces to its complement.

b. For languages A, B if A mapping reduces to B and B mapping reduces to A then $A = B$.

c. For languages A, B if A mapping reduces to B then B mapping reduces to A.

d. Every decidable language mapping reduces to $\emptyset$.

e. Σ^* mapping reduces to every nonempty language over Σ.

Recall mapping reduction is defined in section 5.3: The problem A **mapping reduces** to B means there is a computable function $f : \Sigma^* \to \Sigma^*$ such that for all $x \in \Sigma^*$

$$x \in A \qquad \text{iff} \qquad f(x) \in B.$$

A computable function that makes the "iff" true is said to **witness** the mapping reduction from A to B.

2. **(10 points)** Fix $\Sigma = \{0,1\}$ and $const_{out} \in \Sigma^*$ is a string constant that is not the code of any pair of the form $\langle M, w\rangle$ where M is a Turing machine and w is a string. Consider this computable function:

$F =$ "On input x:

1. If $x \neq \langle M, w\rangle$ for any Turing machine M and string w, output $const_{out}$.
2. Otherwise, let M be the Turing machine and w the string such that $x = \langle M, w\rangle$.
3. Define the Turing machine M' as

   "On input y

   1. Run M on $y^{\mathcal{R}}$. If it accepts, accept. If it rejects, reject. "

5. Output $\langle M', w^{\mathcal{R}}\rangle$."

True/False Briefly justify each answer:

a. For all strings x, if $x \in A_{TM}$ then $F(x) \in HALT_{TM}$.

b. For all strings x, if $F(x) \in HALT_{TM}$ then $x \in A_{TM}$.

c. For all strings x, if $x \in HALT_{TM}$ then $F(x) \in A_{TM}$.

d. For all strings x, if $F(x) \in A_{TM}$ then $x \in HALT_{TM}$.

e. For all strings x, if $x \in A_{TM}$ then $F(x) \in A_{TM}$.

3. **(10 points)** Fix $\Sigma = \{0,1\}$. Consider the following computational problems.

$$N_{TM} = \{\langle M\rangle \mid M \text{ is a Turing machine and } 1 \in L(M)\}$$
$$T2_{TM} = \{\langle M\rangle \mid M \text{ is a Turing machine and } |L(M)| \geq 2\}$$

a. Prove that $T2_{TM}$ mapping reduces to N_{TM}.

b. Prove that N_{TM} mapping reduces to $T2_{TM}$.