
INSTRUCTIONS
This **Individual HW6** must be completed without any collaboration with other students in this class. The only allowed sources of help for this homework are the class textbook, notes, and podcast, and the instructional team.

Your homework **must be typed**. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

READING Sipser Sections 3.2, 3.3, 4.1

KEY CONCEPTS Formal definitions of Turing machines, computations of Turing machines, halting computations, implementation-level descriptions of Turing machines, recognizable languages, decidable languages, decidable computational problems.

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1. (**10 points**) *Extension of Exam 2* In the exam, we considered the following construction: Suppose

$$D = (Q, \Sigma, \delta, q_0, F)$$

is a DFA. We will define a push-down automaton

$$D' = (\ Q \times \{0,1\} \quad \cup \quad \{q_0', q_{acc}'\}\ ,\ \Sigma\ ,\ \Sigma \cup \{\$\}\ ,\ \delta'\ ,\ q_0'\ ,\ \{q_{acc}'\})$$

where we assume $q_0' \notin Q, q_{acc}' \notin Q, \$ \notin \Sigma$ and where δ' is as follows:

$$\begin{array}{rll}
\delta'(\ (q_0', \varepsilon, \varepsilon)\ ) = \{(\ (q_0, 0)\ ,\ \$\ )\} & \\
\delta'(\ (\ (q,0)\ , x, \varepsilon)\ ) = \{(\ (\delta((q,x)), 0)\ ,\ x\ )\} & \text{if } q \in Q, x \in \Sigma \\
\delta'(\ (\ (q,0)\ , \varepsilon, \varepsilon)\ ) = \{(\ (q_0, 1)\ ,\ \varepsilon\ )\} & \text{if } q \in F \\
\delta'(\ (\ (q,1)\ , x, x)\ ) = \{(\ (\delta((q,x)), 1)\ ,\ \varepsilon\ )\} & \text{if } q \in Q, x \in \Sigma \\
\delta'(\ (\ (q,1)\ , \varepsilon, \$)\ ) = \{(\ q_{acc}'\ ,\ \varepsilon\ )\} & \text{if } q \in F \\
\delta'(\ (q, x, y)\ ) = \emptyset & \text{otherwise}
\end{array}$$

For each of the following, answer "Yes", "No", or "Depends on D".
For reference, the correct answers (without justifications) to the exam questions were:
Is $\varepsilon \in L(D')$? Depends on D. Is $L(D')$ decidable? Yes.

a. Is $L(D')$ regular?

b. Is $L(D')$ context-free?

c. Is $\overline{L(D')}$ decidable?

d. Is $L(D') = DOUBLE(L(D))$?

e. Is $L(D') = \{ww^{\mathcal{R}} \mid w \in L(D)\}$?

No justifications required for credit for this question; but, as always, they're a good idea for your own benefit.

2. (**10 points**). Let

$$L_1 = \{a^i b^j \mid 0 \leq j < i\}$$
$$L_2 = \{a^i b^j \mid 0 \leq i < j\}$$

a. Consider the enumerator E_1 given by the high-level description

E_1 = "Ignore the input
1. For integer $i = 0, 1, 2, \ldots$
2. For integer $j = 0, 1, 2, \ldots$
3. If $j > i$, print $a^i b^j$
4. Increment j
5. Increment i"

- List the first three strings printed by E_1.
- Is $L(E_1) = L_1$?
- Is $L(E_1) = L_2$?

b. Consider the enumerator E_2 given by the high-level description

E_2 = "Ignore the input
1. For integer $i = 1, 2, \ldots$
2. For integer $j = 0, 1, 2, \ldots, i - 1$
3. Print $a^i b^j$
4. Increment j
5. Increment i"

- List the first three strings printed by E_2.
- Is $L(E_2) = L_1$?
- Is $L(E_2) = L_2$?

No justifications required for credit for this question; but, as always, they're a good idea for your own benefit.

3. (**10 points**) Consider the following computational problems over a fixed alphabet Σ.

$$A_{DFA} = \{\langle A, w\rangle \mid A \text{ is a DFA over } \Sigma,\ w \in \Sigma^*,\ w \in L(A)\ \}$$
$$E_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA over } \Sigma,\ L(A) = \emptyset\}$$
$$EQ_{DFA} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs over } \Sigma \text{ and } L(A) = L(B)\ \}$$
$$SUB_{DFA} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs over } \Sigma \text{ and } L(A) \subseteq L(B)\ \}$$
$$ALL_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA over } \Sigma,\ L(A) = \Sigma^*\}$$
$$INF_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA over } \Sigma,\ L(A) \text{ is (countably) infinite}\}$$

a. Find all subset relations between distinct sets in this list. That is, determine when $??_{DFA} \subseteq ??_{DFA}$.

b. Find all pairs of sets in this list that are **not** disjoint. That is, determine when $??_{DFA} \cap ??_{DFA} \neq \emptyset$.

No justifications required for credit for this question; but, as always, they're a good idea for your own benefit.