INSTRUCTIONS
This **Individual HW1** must be completed without any collaboration with other students in
this class. The only allowed sources of help for this homework are the class textbook, notes,
and podcast, and the instructional team.

Your homework must be typed. We recommend that you submit early drafts to Gradescope
so that in case of any technical difficulties, at least some of your work is present. You may
update your submission as many times as you'd like up to the deadline.

READING Sipser Section 1.3, 1.1

KEY CONCEPTS Strings, languages, union, concatenation, Kleene star, regular expressions,
language described by a regular expression, deterministic finite automata (DFAs), computa-
tions, language recognized by a DFA.

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1. (**10 points**) For each of the following regular expressions over the alphabet $\{a, b\}$, say whether **each
one** of the following strings is an element of the language described by the regular expression. Justifications
aren't required for credit for this question, but it's good practice to think about how you would explain why
your answer is correct.

   a. Regular expression: $(a \cup b)^*$. Strings: (1) ε (2) ab (3) bba

   b. Regular expression: $aa^* \cup bb^*$. Strings: (1) ε (2) aa (3) ba

   c. Regular expression: $(a \cup b^*) \circ (b \cup a^*)$. Strings: (1) ε (2) a (3) $abba$ (4) bbb

2. (**10 points**) Consider the alphabet $\Sigma = \{0, 1\}$. Define the sets

$L_5 = \{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is a multiple of 5}\}$
$N_5 = \{w \in \{1\}^* \mid \text{the length of } w \text{ is a multiple of 5}\}$
$B_5 = \{w \in \{0, 1\}^* \mid \text{interpreting } w \text{ as a binary number (potentially with leading 0s), } w \text{ is a multiple of 5}\}$

Justifications aren't required for credit for this question, but it's good practice to think about how you would
explain why your answer is correct.

   a. Give an example of a string over Σ that is in L_5 and in N_5 and in B_5, or say None if no such example
   exists.

   b. Give an example of a string over Σ that is in N_5 and not in L_5, or say None if no such example exists.

   c. Give an example of a string over Σ that is in B_5 and not in L_5, or say None if no such example exists.

   d. Give an example of a string over Σ that is in L_5 and in N_5 and not in B_5, or say None if no such
   example exists.

3. (**10 points**) Consider the DFA, M, given by the state diagram:
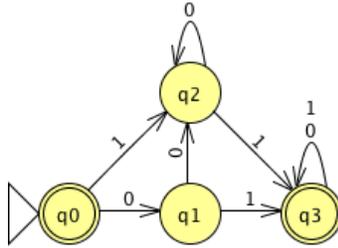
The author of this DFA claims that its formal definition is:

$$M = (\{q0, q1, q2, q3\}, \{0, 1, 2, 3\}, \delta, q0, q3)$$

with δ given by the table

	0	1
$q0$	$q1$	$q2$
$q1$	$q2$	$q3$
$q2$	$q2$	$q3$
$q3$	$q3$	$q3$

a. There are two mistakes in this formal definition. Fix them and write out the corrected formal definition.

b. **True or False** The empty string is accepted by this DFA.

c. **True or False** $L(M)$ is infinite.

d. **True or False** If $x \in L(M)$, the string obtained by flipping bits in x (changing 0 to 1 and 1 to 0) is also in $L(M)$.

e. **True or False** There is string x such that x is in $L(M)$ and $x^{\mathcal{R}}$ is in $L(M)$ as well.

f. **True or False** There is string x such that x is in $L(M)$ but $x^{\mathcal{R}}$ is not in $L(M)$.

Justifications aren't required for credit for this question, but it's good practice to think about how you would explain why your answers are correct.