CSE 105
Homework 3
Due: Sunday April 23, 2017

INSTRUCTIONS

Upload a **single file** to Gradescope for each group. All group members' names and PIDs should be on **each** page of the submission.

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

READING Sipser Sections 1.2, 1.3

KEY CONCEPTS Nondeterministic finite automata (NFA), nondeterministic computation, ε arrows, equivalence of NFA and DFA, regular expressions, equivalence between languages described by regular expressions and languages recognized by automata.

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1. (**8 points**)

(a) **True or False** For every regular language, there is a DFA that has **exactly one accepting state** (i.e. $|F| = 1$) that recognizes the language. Briefly justify your answer.

(b) **True or False** For every regular language, there is an NFA that has **exactly one accepting state** (i.e. $|F| = 1$) that recognizes the language. Briefly justify your answer.

2. (**10 points**) Consider the language

$$L = \{w \in \{a, b, c\}^* \mid w \text{ ends with an } a\}$$

(a) What is L^*? Briefly justify your answer.

(b) Use L as a counterexample to show that the following construction **does not** prove the closure of the set of regular languages under the star operation. That is, you should design an NFA recognizing L and show that the constructed automaton $\hat{N}$ does not recognize L^*. For full credit, include the state diagram for N and for the NFA $\hat{N}$ defined in this construction (briefly justifying each of them), and then explain why $L(\hat{N}) \neq L^*$.

Construction: Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Construct $\hat{N} = (Q, \Sigma, \hat{\delta}, q_0, \hat{F})$, where the states of $\hat{N}$ are the states of N, the alphabet of $\hat{N}$ is the alphabet of N, the start state of $\hat{N}$ is the start state of N, and

– $\hat{F} = \{q_0\} \cup F$ (so that the start state is guaranteed to be an accept state), and

–

$$\hat{\delta}(q, x) = \begin{cases} \delta(q, x) \cup \{q_0\} & \text{if } q \in F \text{ and } x = \varepsilon \\ \delta(q, x) & \text{otherwise} \end{cases}$$

for any $q \in Q$ and $x \in \Sigma_\varepsilon$, so that there is a spontaneous transition from any accepting state back to the initial state.

(c) Use the construction from Theorem 1.49 to construct a NFA recognizing L^*. For full credit, include the state diagram for $\tilde{N}$ based on your diagram for N from part (b).

For reference, here is the construction from page 62 of the textbook.

Construction: Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Construct $\tilde{N} = (\tilde{Q}, \Sigma, \tilde{\delta}, \tilde{q}, \tilde{F})$, the alphabet of $\tilde{N}$ is the alphabet of N, but

– the states of $\tilde{N}$ include a new state $\tilde{Q} = Q \cup \{\tilde{q}\}$,

– the new state is the start state of $\tilde{N}$,

– $\tilde{F} = \{\tilde{q}\} \cup F$ (so that the new start state is guaranteed to be an accept state), and

–

$$\tilde{\delta}(q, x) = \begin{cases} \delta(q, x) \cup \{q_0\} & \text{if } q \in F \text{ and } x = \varepsilon \\ \delta(q, x) & \text{if } q \in F \text{ and } x \in \Sigma \\ \delta(q, x) & \text{if } q \in Q \text{ and } q \notin F \\ \{q_0\} & \text{if } q = \tilde{q} \text{ and } x = \varepsilon \\ \emptyset & \text{if } q = \tilde{q} \text{ and } x \in \Sigma \end{cases}$$

for any $q \in Q$ and $x \in \Sigma_\varepsilon$, so that there is a spontaneous transition from any accepting state back to the initial state.

3. (**8 points**) Let $\Sigma = \{0, 1\}$. Listed below are ten regular expressions, each describing one of five different languages. Pair the equivalent regular expressions and briefly express the language they describe in English. You do not need to justify why the regular expressions describe this language for this question. One pair is done for you as an example.

A. $(10)^* \cup (01)^* \cup (1(01)^*001)$

B. $(0 \cup 1)^*$

C. $0(10)^*1 \cup \varepsilon$

D. $((01)^* \cup (10)^*)(01 \cup \varepsilon)$

E. $(01)^* \cup 01(01)^*01$

F. $(0^*10)^*$

G. $(0 \cup 10)^*10 \cup \varepsilon$

H. $0^*(0^*10^*10^*)^*$

I. $(0^*1^*)^*$

J. $(0^*10^*1)^*(0 \cup \varepsilon)^*$

Example B. and I. both represent the same language, the language of **all** strings over $\{0, 1\}$.

4. (**8 points**) Find a DFA recognizing the **complement** of the language described by the regular expression over $\{a, b\}$

$$a(abb)^* \cup b$$

For full credit, use the procedure given in Theorem 1.54 to convert this regular expression to an NFA. Then, convert the NFA to an equivalent DFA, and then form the DFA that recognizes the complement. Include and label the state diagrams for each of these intermediate machines.

5. (**1 point**) As part of a multi-year study, certain components of this course are being studied for their effectiveness in improving the learning environment and outcomes of this class. Please indicate whether you consent to have your anonymized data included in the study analysis.

Consent form: `https://goo.gl/forms/Rw4rQinyK5ukZxbO2`

You will receive credit for indicating that you completed the consent form, whether or not you choose to participate in the study. Just as in homework 1, we're using the honor system: to declare that you have read an completed the form, simply include the statement "I completed the consent form" (if you're working individually) or "Each group member completed the consent form" (if you're working in a group) as this homework question's answer.