

p.220, icon at Example 3

#4. Find values a , b , and c (not all 0) such that $(abc)_5 = (cba)_8$, or prove that there are none.

Solution:

Note that each of a , b , and c must be between 0 and 4 because the base of the number on the left is 5. Expanding $(abc)_5$ and $(cba)_8$ in terms of base 5 and 8 respectively yields

$$(abc)_5 = 25a + 5b + c \text{ and } (cba)_8 = 64c + 8b + a.$$

If $(abc)_5 = (cba)_8$, then

$$25a + 5b + c = 64c + 8b + a,$$

or

$$24a - 3b - 63c = 0.$$

This simplifies to

$$8a - b - 21c = 0.$$

The only solution with each variable between 0 and 4 (and not all 0) is $a = b = 3$ and $c = 1$. (This is easily seen by trial and error.) Hence $(331)_5 = (133)_8 = 91$.
