CSE 20
DISCRETE MATH

SPRING 2016

http://cseweb.ucsd.edu/classes/sp16/cse20-ac/

Today's learning goals

- Determine the truth value of predicates for specific values of their arguments
- Define the universal and existential quantifiers
- Translate sentences from English to predicate logic using appropriate predicates and quantifiers
- Use appropriate Boolean operators to restrict the domain of a quantified statement
- Negate quantified expressions
- Translate quantified statements to English, even in the presence of nested quantifiers
- Evaluate the truth value of a quantified statement with nested quantifiers

Quantifiers

"P(x) for all values x in the domain"

$$\forall x P(x)$$

"There exists an element in the domain such that P(x)"

$$\exists x P(x)$$

Translations

Rosen p. 54 #28

Something is not in the correct place.

Everything is in the correct place and in excellent condition.

All tools are in the correct place and are in excellent condition.

Nothing is in the correct place and is in excellent condition.

One of the tools is not in the correct place, but it is in excellent condition.

Translations

Rosen p. 54 #28

Something is not in the correct place.

$$\exists x \neg P(x)$$

Everything is in the correct place and in excellent condition.

$$\forall x (P(x) \wedge D(x))$$

All tools are in the correct place and are in excellent condition.

$$\forall x (T(x) \rightarrow (P(x) \wedge D(x)))$$

Nothing is in the correct place and in excellent condition.

One of the tools is not in the correct place, but it is in excellent condition.

Transla

Something is

Everything is

All tools are

Nothing is in the correct place and in excellent condition.

One of the tools is not in the correct place, but it is in excellent condition.

A. $\forall x(\neg P(x) \wedge \neg D(x))$

B. $\neg\forall x(P(x) \wedge D(x))$

C. $\neg\exists x(P(x) \wedge D(x))$

D. $\exists x(\neg P(x) \wedge \neg D(x))$

E. None of the above.

"De Morgan"-ish

Rosen p. 45

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$
$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

Logical equivalence of quantified statements: no matter which predicates are substituted and no matter which domains of discourse, the statements have the same truth value

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$$\forall x(P(x) \wedge D(x))$$

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$$\forall x(T(x) \rightarrow (P(x) \wedge D(x)))$$

Nothing is in the correct place and is in excellent condition.

$\neg\exists x(P(x) \wedge D(x))$, or equivalently $\forall x(\neg P(x) \vee \neg D(x))$

One of the tools is not in the correct place, but it is in excellent condition.

Translations

Rosen p. 54 #28

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One of the tools is not in the correct place, but it is in excellent condition.

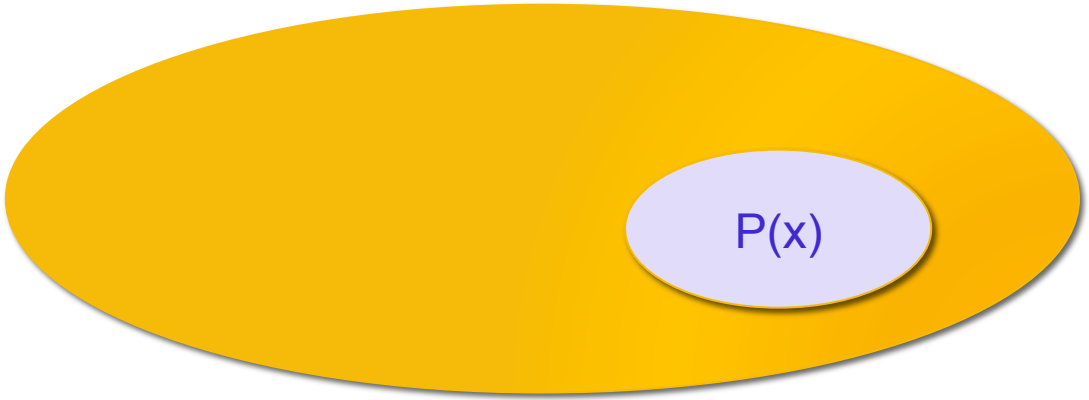
$$\exists x(\ T(x) \wedge \neg P(x) \wedge D(x)\ )$$

Restricting the domain

Rosen p. 44

$$\forall x(P(x) \rightarrow Q(x))$$

$$\exists x(P(x) \wedge Q(x))$$



Restricting the domain

Rosen p. 44

$$\forall x(P(x) \rightarrow Q(x))$$

$$\exists x(P(x) \wedge Q(x))$$

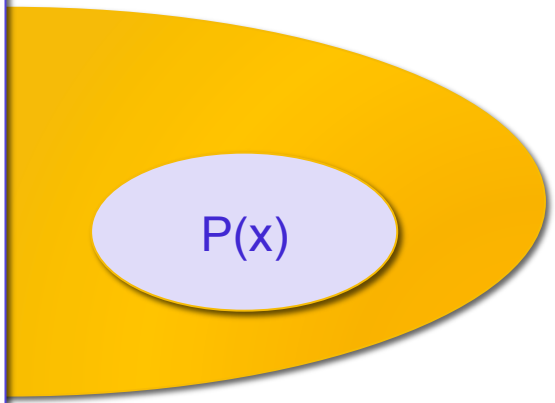
What's the negation of $\forall x(P(x) \rightarrow Q(x))$?

A. $\forall x(P(x) \wedge \neg Q(x))$

B. $\forall x(P(x) \rightarrow \neg Q(x))$

C. $\exists x(P(x) \wedge \neg Q(x))$

D. $\exists x(P(x) \rightarrow \neg Q(x))$

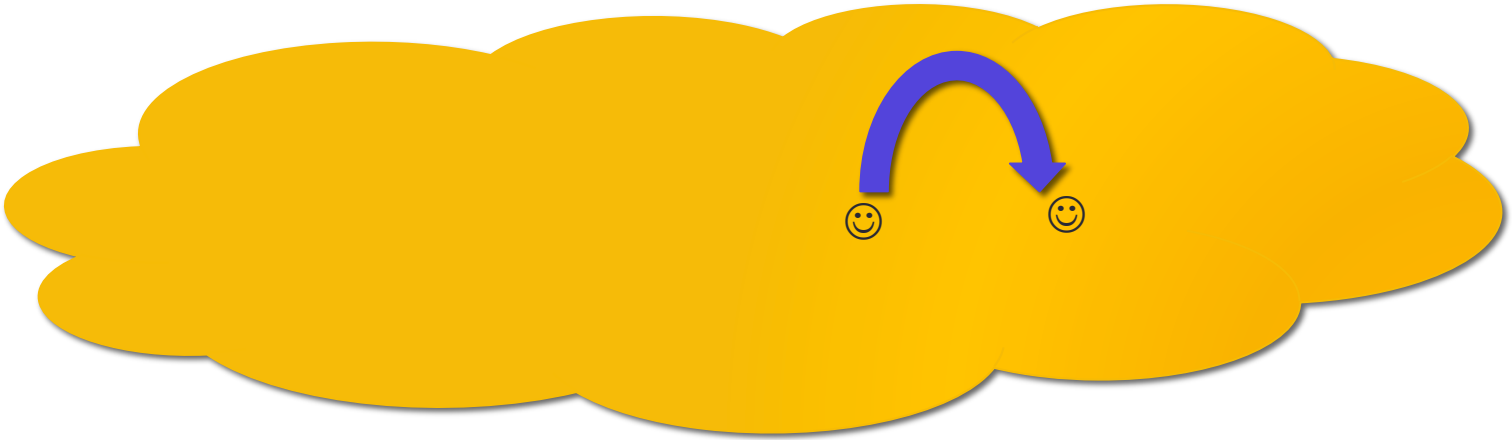


Nested quantifiers

Rosen p. 64 #3

Q(x,y) "x has send a text to y" *domain: students in class*

$$\exists x \exists y Q(x, y)$$



Nested quantifiers

Rosen p. 64 #3

Q(x,y) "x has send a text to y" *domain: students in class*

$$\exists x \forall y Q(x, y)$$

Nested quantifiers

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Nested quantifiers

Rosen p. 64 #3

Q(x,y) "x has send a text to y" *domain: students in class*

$$\forall x \forall y Q(x, y)$$

Evaluating quantified statements

Rosen p. 64 #1

$$\forall x \exists y (x < y)$$

In which domain is this statement true?

A. All real numbers.
B. All positive real numbers.
C. All positive integers.
D. All real numbers in closed interval [0,1].
E. The integers 1,2,3.

Evaluating quantified statements *Rosen p. 64 #1*

$$\forall x \forall y(((x \geq 0) \wedge (y \geq 0)) \rightarrow (xy \geq 0))$$

In which domain is this statement true?

A. All real numbers.
B. All positive real numbers.
C. All positive integers.
D. All real numbers in closed interval [0,1].
E. The integers 1,2,3.

Evaluating quantified statements

Rosen p. 64 #1

$$\forall x \forall y \exists z (xy = z)$$

In which domain is this statement true?

A. All real numbers.
B. All positive real numbers.
C. All positive integers.
D. All real numbers in closed interval [0,1].
E. The integers 1,2,3.

And in the other direction … *Rosen p. 66 #23*

- "The product of two negative real numbers is positive."

And in the other direction … *Rosen p. 66 #23*

- "The difference of a real number and itself is zero."

And in the other direction … *Rosen p. 66 #23*

- "A negative real number does not have a square root that is a real number."

And in the other direction … *Rosen p. 66 #23*

- "Every positive real number has exactly two square roots."

Logical equivalence

Rosen p. 646 #23

Is it true that for every meaning of the predicate and every domain of discourse

$\neg\exists x\forall y P(x,y)$ and $\forall x\exists y\neg P(x,y)$

have the same truth value?

Next up

- Rules of inference and proof strategies!