CSE 20
DISCRETE MATH

SPRING 2016

http://cseweb.ucsd.edu/classes/sp16/cse20-ac/

Today's learning goals

- Convert between positive integers written in any base
- Relate algorithms for integer operations to bitwise boolean operations
- Correctly use XOR and bit shifts

About you

CENTR101: CA PCYNH109: AB

To change your remote frequency

1. Press and hold power button until flashing
2. Enter two-letter code
3. Checkmark / green light indicates success

How many people were in your group for HW1?

A. I worked alone.
B. 2
C. 3
D. I joined this class late and didn’t submit HW1.

Recall: Base expansion

Rosen p. 246

Notation: for positive integer n

Write

$$\boxed{(a_k\ a_{k-1}\ \ldots\ a_1\ a_0)_b}$$

when

$$n = a_k b^k + a_{k-1} b^{k-1} + \ \ldots + a_1 b + a_0$$

Base b expansion of n

What's the base 2 expansion of 42?

A. 111111
B. 100001
C. 101010
D. 110011
E. None of the above

What's the base 2 expansion of 42?

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Some tricks!

What's the **biggest** integer value whose binary representation has 4 bits?

A. $2^4 = 16$
B. $2^3 = 8$
C. 4
D. 1000
E. None of the above

What's the **smallest** integer value whose binary representation has 4 bits?

A. 0
B. 1
C. 8
D. 1000
E. None of the above

Fixed width "binary expansions" with 4 bits

$(0000)_2 = 0$

$(0001)_2 = 1$

$(0010)_2 = 2$

…

$(1110)_2 = 14$

$(1111)_2 = 15$

What about negative numbers?

From HW2… Two's complement

Use n bits to represent integers in the range **$[-2^{n-1}, 2^{n-1} -1]$**

From HW2… Two's complement

Use n bits to represent integers in the range $[-2^{n-1}, 2^{n-1}-1]$

e.g. for 4 bits the range is -8, -7, -6, … , 5, 6, 7.

0000 → 0

n-1 rather than n because use 1 bit to signal positive vs. negative

From HW2… Two's complement

Use n bits to represent integers in the range $[-2^{n-1}, 2^{n-1} -1]$

e.g. for 4 bits the range is -8, -7, -6, … , 5, 6, 7.

..

0010 → 2

0001 → 1

0000 → 0

From HW2… Two's complement

Use n bits to represent integers in the range $[-2^{n-1}, 2^{n-1} -1]$

e.g. for 4 bits the range is -8, -7, -6, … , 5, 6, 7.

..

0010 → 2

0001 → 1

0000 → 0

Use leading 0 to indicate positive.

From HW2… Two's complement

Use n bits to represent integers in the range $[-2^{n-1}, 2^{n-1} -1]$

e.g. for 4 bits the range is -8, -7, -6, … , 5, 6, 7.

..

0010 → 2

0001 → 1

0000 → 0

For negative integers x the leftmost bit is 1 and the remaining n-1 bits are the binary expansion of

$$2^{n-1} - |x|$$

From HW2… Two's complement

Use n bits to represent integers in the range $[-2^{n-1}, 2^{n-1} - 1]$

e.g. for 4 bits the range is -8, -7, -6, … , 5, 6, 7.

..

0010 → 2

0001 → 1

0000 → 0

For negative integers x the leftmost bit is 1 and the remaining n-1 bits are the binary expansion of $2^{n-1} - |x|$

What is the 4 bit two's complement representation of -1?

A. 0001
B. 1001
C. 1111
D. 0111
E. None of the above

From HW2… Two's complement

Use n bits to represent integers in the range $[-2^{n-1}, 2^{n-1} -1]$

e.g. for 4 bits the range is -8, -7, -6, … , 5, 6, 7.

..

0010 → 2

0001 → 1

0000 → 0

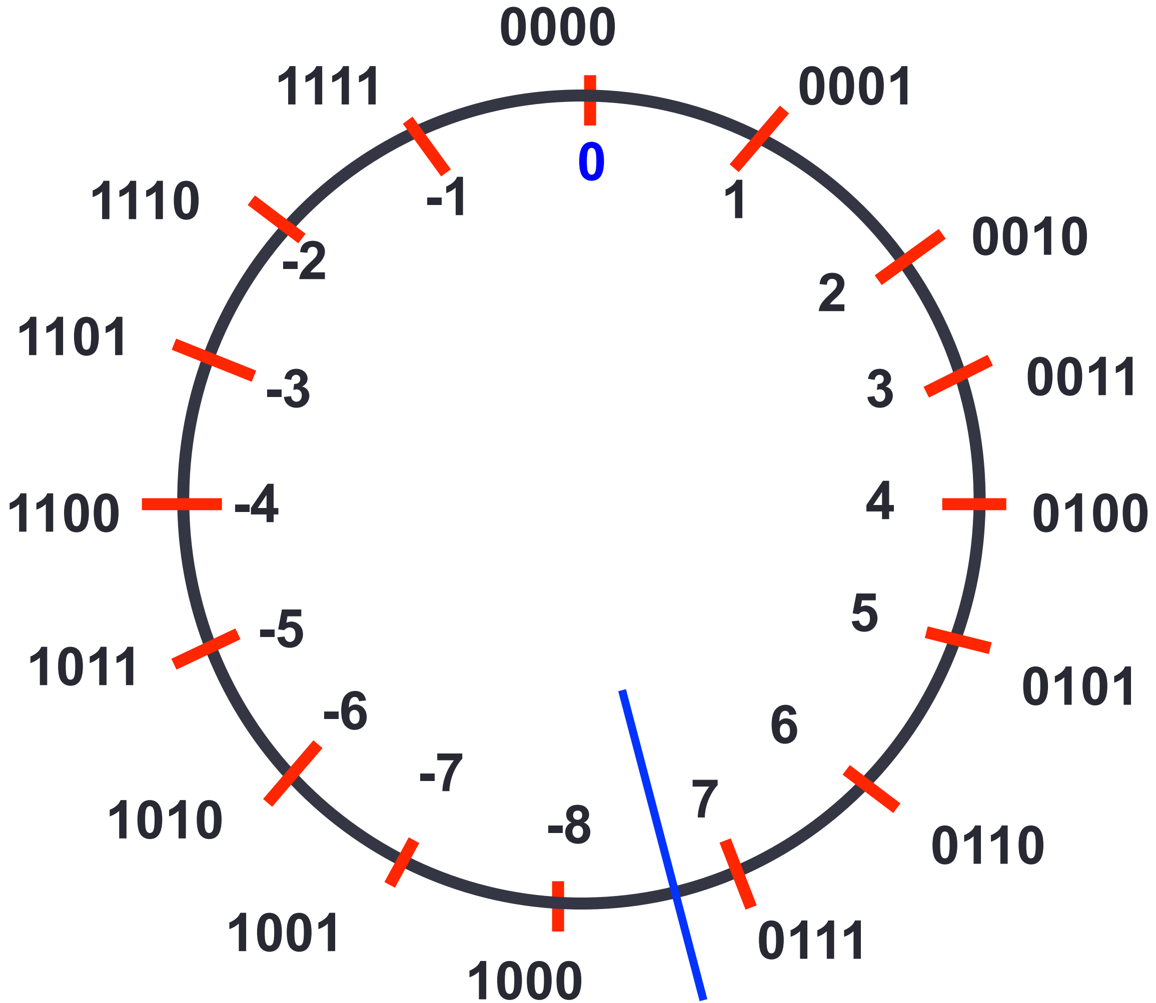
1111 → -1

1110 → -2

…

For positive integers x
the leftmost bit is 0 and the remaining n-1 bits are the binary expansion of x

For negative integers x
the leftmost bit is 1 and the remaining n-1 bits are the binary expansion of
$$2^{n-1} - |x|$$


0000
0
0001
1
0010
2
0011
3
0100
4
0101
5
0110
6
0111
7
1000
-8
1001
-7
1010
-6
1011
-5
1100
-4
1101
-3
1110
-2
1111
-1

Adding

What's the result of adding 2 and -2?

..

0010	→	2
0001	→	1
0000	→	0
1111	→	-1
1110	→	-2

…

Adding

What's the result of adding 2 and -2?

Should equal 0

..

0010 → 2

0001 → 1

0000 → 0

1111 → -1

1110 → -2

…

$$\begin{array}{r} 0010 \\ +\ 1110 \\ \hline 10000 \end{array}$$

Adding

Should equal 0

What's the result of adding 2 and -2?

..

0010 → 2

0001 → 1

0000 → 0

1111 → -1

1110 → -2

...

$$
\begin{array}{r}
0010 \\
+\ \underline{1110} \\
10000
\end{array}
$$

Key: to zero out bits, each column needs to cancel.

Arithmetic + Representations

Rosen p. 251,826

In base b,

$$\begin{array}{r} s_{k-1}\ \ldots\ s_1\ s_0 \\ +\ t_{k-1}\ \ldots\ t_1\ t_0 \\ \hline \end{array}$$

Basic operations: one symbol addition, carry

Arithmetic + Representations

Rosen p. 251,826

For decimal

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4										
5										
6										
7										
8										
9										

Arithmetic + Representations

Rosen p. 251,826

For binary

	0	1
0	0	1
1	1	0

Arithmetic + Representations

Rosen p. 251,826

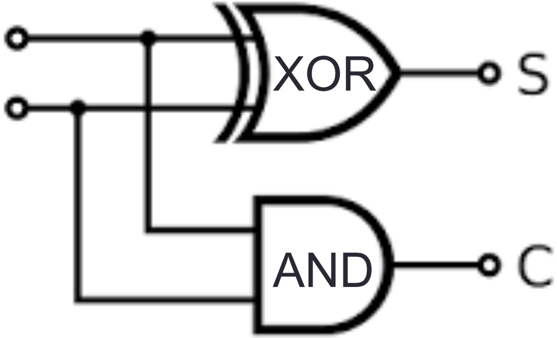
Alternatively,

Input		Output	
x	**y**	**Sum**	**Carry**
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Arithmetic + Representations

Rosen p. 251,826

Alternatively,



Input		Output	
x	**y**	**Sum**	**Carry**
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Half adder logic circuit

Computer bit operations *Rosen p. 11*

x	y	$x \vee y$ x OR y	$x \wedge y$ x AND y	$x \oplus y$ x XOR y
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

"x OR y is 1 if at least one of x or y is 1"

"x AND y is 1 if both x and y are 1"

"x XOR y is 1 if exactly one of x and y is 1"

Computer bit operations

Rosen p. 11

x	y	$x \vee y$ x OR y	$x \wedge y$ x AND y	$x \oplus y$ x XOR y
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

What is the **bitwise** AND of the strings 0011 and 0101?

A. 0001

B. 0111

C. 0010

D. 0100

E. None of the above

Computer bit operations

Rosen p. 11

x	y	x ∨ y x OR y	x ∧ y x AND y	x y x XOR y
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

What is the **bitwise** XOR of the strings 0011 and 0101?

A. 0110

B. 0000

C. 0111

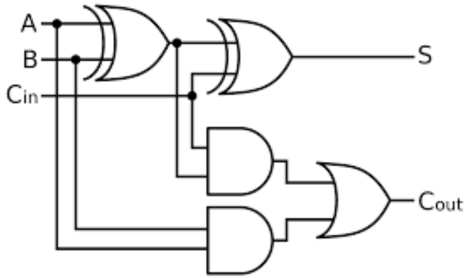
D. 0100

E. None of the above

Logic

Rosen Section 1.1

- Use gates and circuits to express arithmetic.



- Precisely express theorems and invariant statements.
- Make valid arguments to prove theorems.

Definitions

Rosen p. 2-4

- **Proposition**: declarative sentence that is T or F (not both)
- **Propositional variable**: variables that represent propositions.
- **Compound proposition**: new propositions formed from existing propositions using logical operators.
- **Truth table**: table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Truth tables

How many rows are in the truth table for **p v q**, the disjunction of p and q?

A. 0110
B. 0000
C. 0111
D. 0100
E. None of the above

Truth tables

- Specify logical operator by truth table.
- Can use truth table to compute value of compound variable.

- *Next time: how to prove two tables are equivalent?*

Reminders

- Homework 2 due Friday
  - Integer representations
  - Algorithms
- Office hours