CSE 20
DISCRETE MATH

SPRING 2016

http://cseweb.ucsd.edu/classes/sp16/cse20-ac/

Today's learning goals

- Define and differentiate between important sets
- Use correct notation when describing sets: {...}, intervals
- Define and prove properties of: subset relation, power set, Cartesian products of sets, union of sets, intersection of sets, disjoint sets, set differences, complement of a set
- Describe computer representation of sets with bitstrings
- Define and compute the cardinality of finite sets

Some definitions

Rosen Sections 2.1, 2.2

Set: unordered **collection** of elements

Arrows in set builder notation indicate "and"

N: natural numbers {0, 1, 2, 3, …}

Z: integers {…, -2, -1, 0, 1, 2, …}

Z^+: positive integers {1, 2, 3, …}

Q: rational numbers $\{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$

R: real numbers

R^+: positive real numbers

C: complex numbers

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$

Theorem: **Z** is a subset of **Q**.

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$

How would you prove that **R** is not a subset of **Q**?

A. Prove that every real number is not rational.
B. Prove that every rational number is real.
C. Prove that there is a real number that is rational.
D. Prove that there is a real number that is not rational.
E. Prove that there is a rational number that is not real.

An (ir)rational excursion

Rosen p. 86, 97

Theorem: R is not a subset of **Q**.

Lemma: $\sqrt{2}$ is not rational.

Corollary: There are irrational numbers x,y such that x^y is rational.

An (ir)rational excursion

Rosen p. 86, 97

Theorem: R is not a subset of **Q**.

What's the logical structure of each of these statements?

Lemma: $\sqrt{2}$ is not rational.

Recall: **Q**=$\{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$

Details of proof in page 86: use contradiction!

Corollary: There are irrational numbers x,y such that x^y is rational.

Existential statement: can we build a witness?

An (ir)rational excursion

Rosen p. 86, 97

Theorem: R is not a subset of **Q**.

What's the logical structure of each of these statements?

Lemma: $\sqrt{2}$ is not rational.

Corollary: There are irrational numbers x,y such that x^y is rational.

Some definitions

Rosen Sections 2.1, 2.2

Empty set: $\emptyset = \{\} = \{x : x \neq x\}$

Which of the following is not equal to the rest?

A. $\{\}$

B. $\{\emptyset\}$

C. $\emptyset$

D. $\{x \in \mathbb{Z} \mid x > x^2\}$

E. $\{x \mid x \in \emptyset\}$

Operations on sets

Rosen Sections 2.1, 2.2

Power set: For a set S, its power set is the set of all subsets of S. $\mathcal{P}(S) = \{A \mid A \subseteq S\}$

Which of the following is **not** true (in general)?

A. $S \in \mathcal{P}(S)$

B. $\emptyset \in \mathcal{P}(S)$

C. $S \subseteq \mathcal{P}(S)$

D. $\emptyset \subseteq \mathcal{P}(S)$

E. $\emptyset \in S$

Operations on sets

Rosen Sections 2.1, 2.2

- Given two sets A, B we can define

$A \cap B$ **Intersection of A and B**

$A \cup B$ **Union of A and B**

$A - B$ **Difference of A and B**

$A \times B$ **Cartesian product of A and B**

Operations on sets

Rosen Sections 2.1, 2.2

- Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Operations on sets

Rosen Sections 2.1, 2.2

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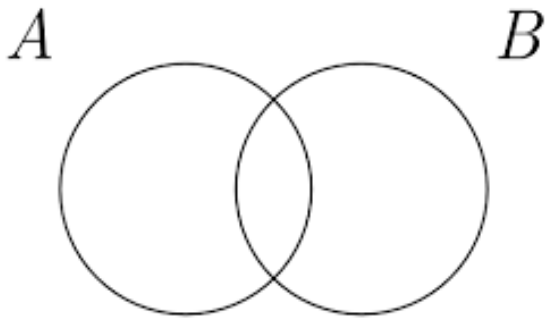


Which of the following **can't** be labelled in this Venn diagram?

A. $A \cap B$
B. $A \cup B$
C. $A - B$
D. $A \times B$
E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

- Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Which of these is true?

A. $A \cap B = B \cap A$

B. $A \cup B = B \cup A$

C. $A - B = B - A$

D. $A \times B = B \times A$

E. None of the above.

Sizes of sets

- If S is a set with exactly *n* **distinct** elements, with *n* a nonnegative integer, then S is **finite set** and |S| = n.

Which of the following sets are finite?
Assume universe is set of real numbers.

A. $\emptyset$
B. $\mathbb{Q} \cap \{x \mid 0 \leq x \leq 1\}$
C. $\mathbb{Z} \cap \{x \mid 0 \leq x \leq 1\}$
D. $\mathbb{Z} \cup \{x \mid 0 \leq x \leq 1\}$
E. None of the above.

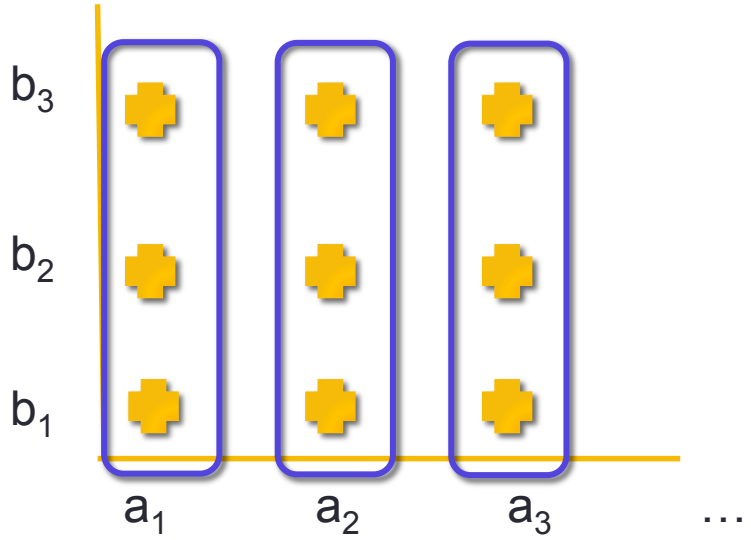
Operations on sets

Rosen Sections 2.1, 2.2

- If the sets A, B are finite then

$$|A \times B| = |A| \cdot |B|$$

|B| many elements for each of the |A| many elements in A

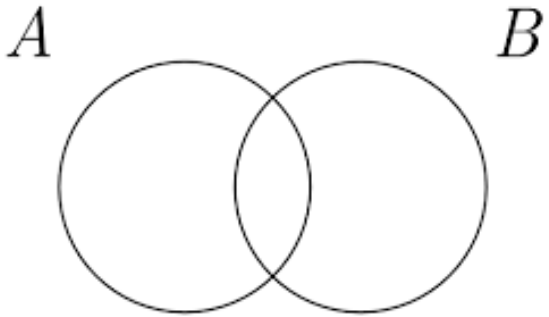


Operations on sets

Rosen Sections 2.1, 2.2

- If the sets A, B are finite then

$$|A \cup B| = ?$$



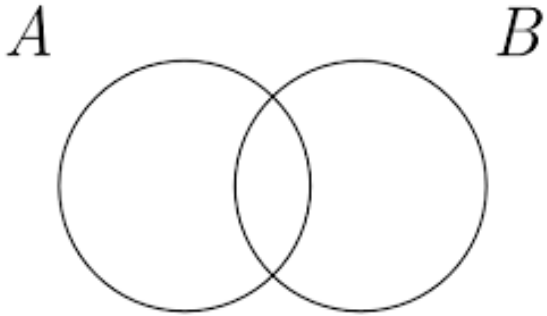
A. |A| + |B|
B. |A| - |B|
C. |A| |B|
D. $|A|^{|B|}$
E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

- If the sets A, B are finite then

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Operations on sets

Rosen Sections 2.1, 2.2

- If the sets A, B are finite then

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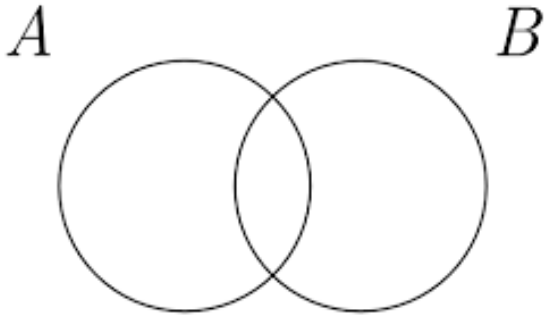


What's the size of the difference A-B?

A. $|A \cup B| - |A \cap B|$

B. $|A \cup B| - |A| - |B|$

C. $|A| - |A \cap B|$

D. $|A| - |B|$

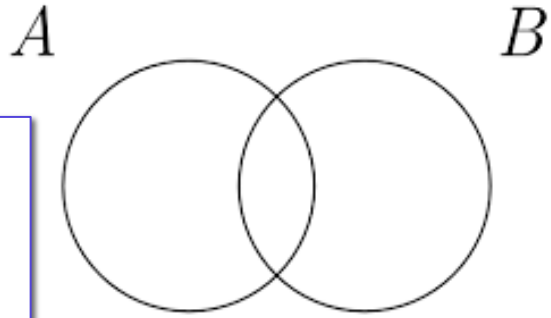
E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

- Two sets A and B are **disjoint** iff

$$A \cap B = \emptyset$$



Which of the following is **not** an equivalent characterization of A and B being disjoint?

A. |A U B| = |A| + |B|
B. $A - B = A$
C. $A \subset B$
D. $\neg\exists x(x \in A \wedge x \in B)$
E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

- If the set A is finite then

$$|\mathcal{P}(A)| = ?$$

*Does the power set of A depend **just** on the size of A?*

*Does the **size** of the power set of A depend **just** on the size of A?*

Operations on sets

Rosen Sections 2.1, 2.2

- If the set A is finite then

$$|\mathcal{P}(A)| = ?$$

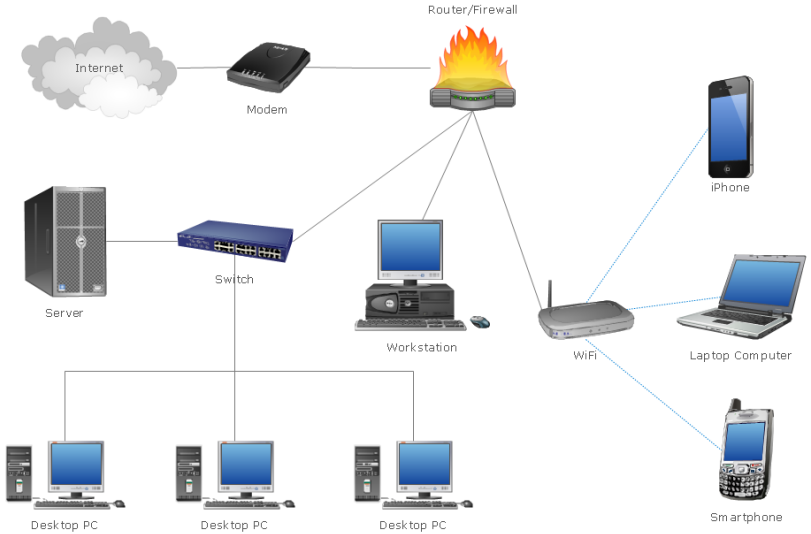
*Does the power set of A depend **just** on the size of A?*

*Does the **size** of the power set of A depend **just** on the size of A?*

$$\begin{aligned}
\mathcal{P}(\{2\}) &= \{\emptyset, \{2\}\} \\
\mathcal{P}(\{8\}) &= \{\emptyset, \{8\}\} \\
\mathcal{P}(\{2,5\}) &= \{\emptyset, \{2\}, \{5\}, \{2,5\}\} \\
\mathcal{P}(\{2,8\}) &= \{\emptyset, \{2\}, \{8\}, \{2,8\}\} \\
\mathcal{P}(\{2,5,8\}) &= \{\emptyset, \{2\}, \{5\}, \{8\}, \{2,5\}, \{2,8\}, \{5,8\}, \{2,5,8\}\}
\end{aligned}$$

Representing sets

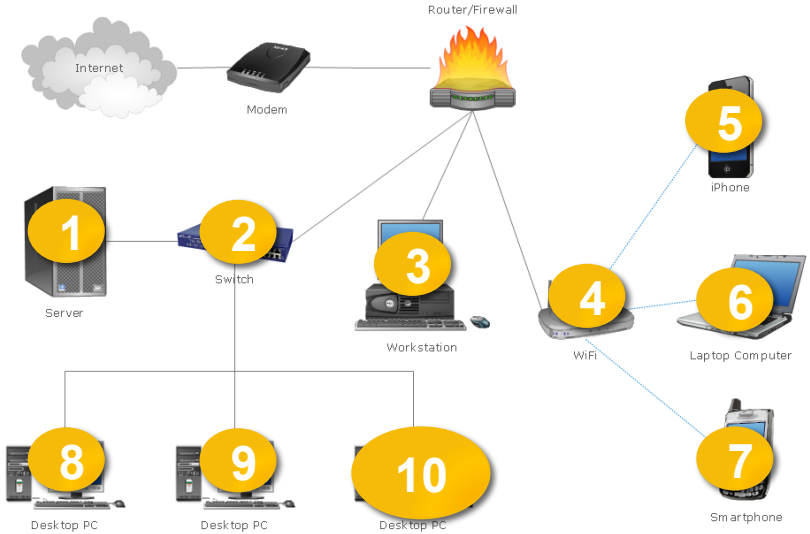
Rosen p. 134



Set of home network components:
{ server, switch, workstation, wifi, iPhone, laptop, Smartphone, Desktop Roommate1, Desktop Roommate2, Desktop Roommate3}

Representing sets

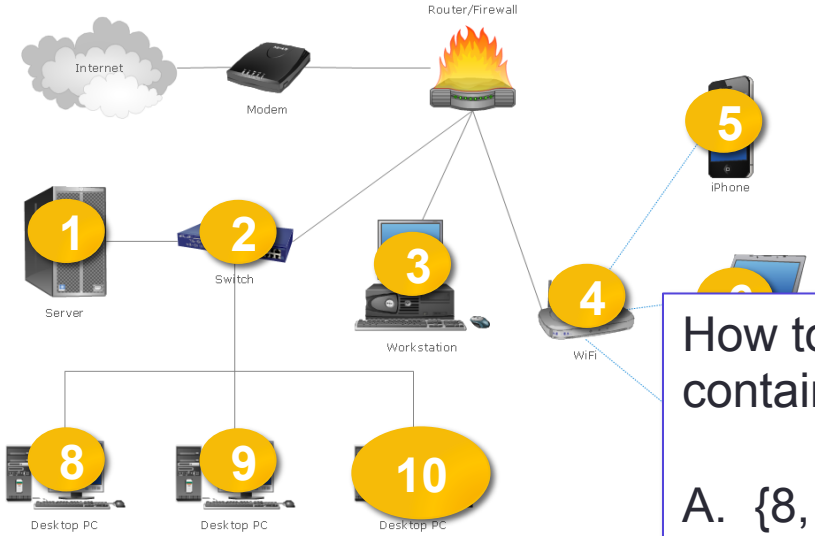
Rosen p. 134



Set of home network components:
{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Representing sets

Rosen p. 134



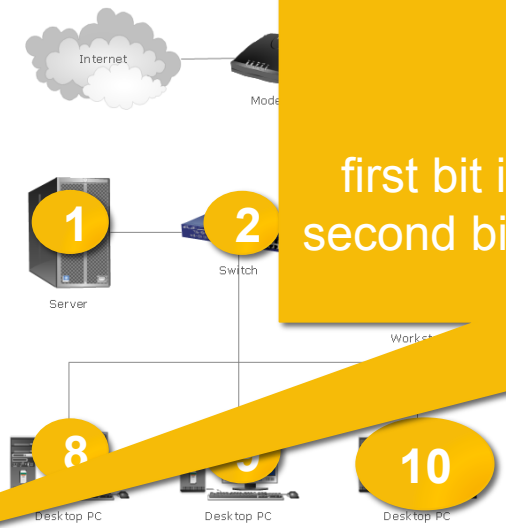
Set of home network components:
{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

How to represent the subset which contains all the desktop PCs?

A. {8, 9, 10}
B. {10, 8, 9}
C. {8, 8, 9, 10}
D. None of the above.
E. All of the above.

Representing sets

Rosen p. 134



Alternatively: using **bit strings**

– – – – – – – – – – –

first bit is 0 if item 1 (server) is not in the set; 1 if is
second bit is 0 if item 2 (switch) is not in the set; 1 if is

Set of home network components:
{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

How to represent the subset which contains all the desktop PCs **using bit strings**?

A. 0000000000
B. 0000000111
C. 1110000000
D. None of the above.
E. All of the above.

Representing sets

Rosen p. 134

Assume that universal set U is finite of size *n* (and n is not too big).
Specify arbitrary ordering of elements of U: a_1, a_2, a_3, …, a_n .

Represent subset A of U by the bit string where, for each i, the *i*th bit in the bit string is 1 if a_i is an element of A and it's 0 if a_i is not an element of A.

Describe, using set operations, the set described by the bit string which results from taking bit string for A and flipping each bit 0 → 1, 1 → 0.

A. The power set of A, $\mathcal{P}(A)$
B. The union of A with iself, $A \cup A$
C. The difference U – A
D. The Cartesian product $A \times A$
E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

- If the set A is finite then

$$|\mathcal{P}(A)| = ?$$

How many subsets of A are there?

Represent subset A of U by the bit string where, for each i, the *i*th bit in the bit string is 1 if a_i is an element of A and it's 0 if a_i is not an element of A.

How many different substrings are there?

Operations on sets

Rosen Sections 2.1, 2.2

- If the set A is finite then

$$|\mathcal{P}(A)| = 2^{|A|}$$

How many subsets of A are there?

Represent subset A of U by the bit string where, for each i, the *i*th bit in the bit string is 1 if a_i is an element of A and it's 0 if a_i is not an element of A.

How many different substrings are there?

Next up

- How do we prove these general formulas?
- Induction!