Model Fitting

Introduction to Computer Vision

CSE 152

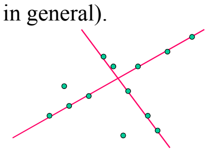
Lecture 11

Announcements

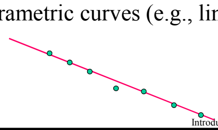
- Graded homework 1 was returned last week
- Homework 2 is due May 7, 11:59 PM
  - Extended three days
- Reading:
  - Chapter 10: Grouping and Model Fitting

What to do with edges?

- Segment linked edge chains into curve features (e.g., line segments).
- Group unlinked or unrelated edges into lines (or curves in general).



- Accurately fitting parametric curves (e.g., lines) to grouped edge points.



Hough Transform
[Patented 1962]

Finding lines in an image

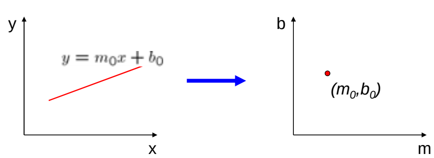


image space Hough space

Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space

Finding lines in an image

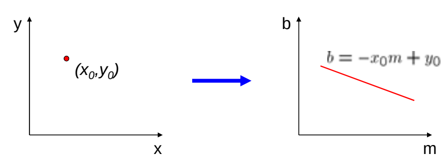


image space Hough space

Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- What does a point (x_0, y_0) in the image space map to?

  The equation of any line passing through (x_0, y_0) has form

  $y_0 = mx_0 + b$

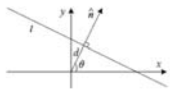
  This is a line in Hough space: $b = -x_0 m + y_0$

Hough Transform Algorithm

- Typically use a different parameterization

  $d = x cos\theta + y sin\theta$

  - d is the perpendicular distance from the line to the origin
  - θ is the angle this perpendicular makes with the x axis



- Basic Hough transform algorithm
  1. Initialize H[d, θ]=0 ; H is called accumulator array
  2. for each edge point I[x,y] in the image

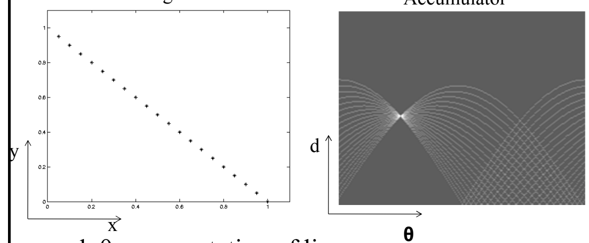
     for θ = 0 to 180

     $d = x cos\theta + y sin\theta$

     H[d, θ] += 1
  3. Find the value(s) of (d, θ) where H[d, θ] is the global maximum
  4. The detected line in the image is given by $d = x cos\theta + y sin\theta$
- What's the running time (measured in # votes)?

Hough Transform: 20 colinear points

Image Accumulator

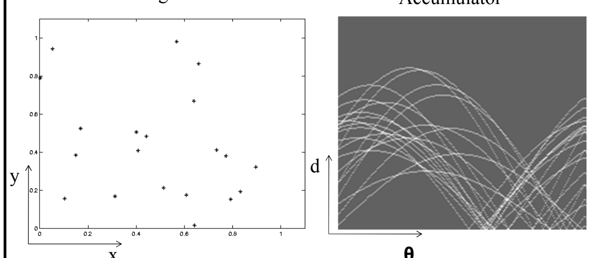


- d, θ representation of line
- Drawn as: $d = |x cos\theta + y sin\theta|$
- Maximum accumulator value is 20

Note: accumulator array range: theta: 0-360, d: positive

Hough Transform: Random points

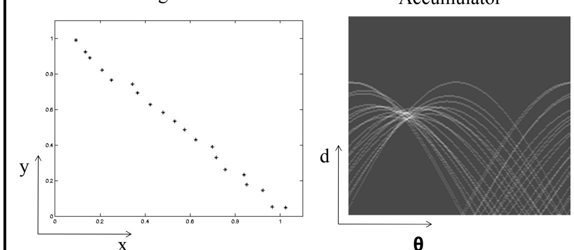
Image Accumulator



- d, θ representation of line
- Drawn as: $d = |x cos\theta + y sin\theta|$
- Maximum accumulator value is 4

Hough Transform: "Noisy line"

Image Accumulator



- d, θ representation of line
- Drawn as: $d = |x cos\theta + y sin\theta|$
- Maximum accumulator value is 6

Extension: Oriented Edges

procedure *Hough*($\{(x, y, \theta)\}$):

1. Clear the accumulator array.
2. For each detected edgel at location (x, y) and orientation $\theta = \tan^{-1} n_y/n_x$, compute the value of

   $$d = x n_x + y n_y$$

   and increment the accumulator corresponding to (θ, d).
3. Find the peaks in the accumulator corresponding to lines.
4. Optionally re-fit the lines to the constituent edgels.

Algorithm 4.2 Outline of a Hough transform algorithm based on oriented edge segments.

Hough Transform for Curves

Generalized Hough Transform

- The Hough transform can be generalized to detect any curve that can be expressed in parametric form:

  $y = f(x, a_1, a_2, \ldots a_p)$

  or

  $g(x, y, a_1, a_2, \ldots a_p) = 0$

  - $a_1, a_2, \ldots a_p$ are the parameters
  - The parameter space is p-dimensional
  - The accumulating array is *large*

Example: Finding circles

Equation for circle is

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

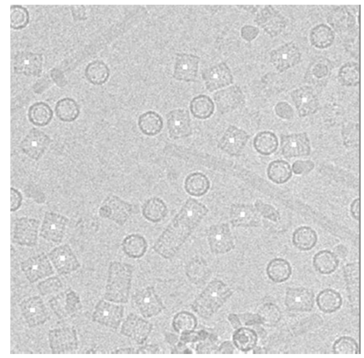
where the parameters are the circle's center (x_c, y_c) and radius r.

Three dimensional generalized Hough space.

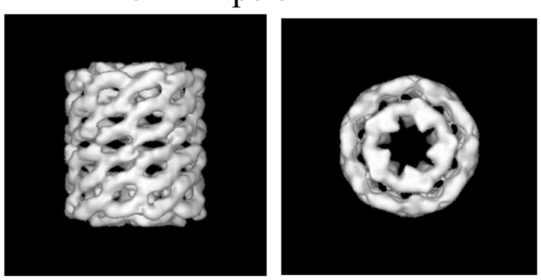
Given an edge point (x,y),

1. Loop over all values of (x_c, y_c),
2. Compute r
3. Increment $H(x_c, y_c, r)$

Transmission Electron Microscopy (TEM) Image of Keyhole Limpet Hemocyanin (KLH) with detected particles



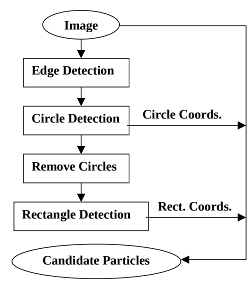
3D Maps of KLH



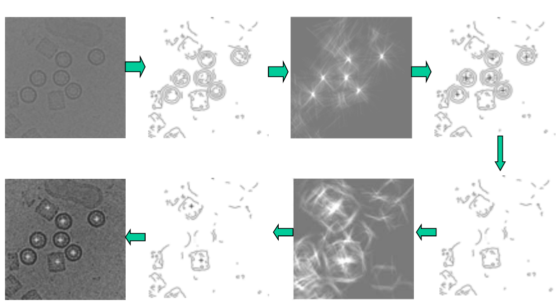
Three-dimensional maps of KLH at a resolution of 23.5 Å reconstructed using particles extracted either manually or automatically as described in the text. (a), (b) The side- and top- view of a 3D map reconstructed from a set of 1042 manually selected particle images.

Processing in Stage 1 for KLH

- Canny edge detection.
- A sequence of ordered Hough transforms (HTs) is applied in order from the computationally simplest one to the most complex one.
- Edges covered by the detected shapes are removed immediately from edge images following the application of the last HT.



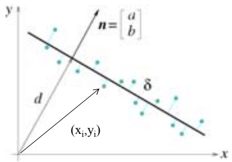
Picking KLH Particles in Stage 1



Zhu et al., IEEE Transactions on Medical Imaging, 2003

Line Fitting

Line Fitting



Given n points (x_i, y_i), estimate parameters of line

$$ax_i + by_i - d = 0$$

subject to the constraint that

$$a^2 + b^2 = 1$$

Note: $ax_i + by_i - d$ *is distance from* (x_i, y_i) to line.

Problem: minimize

$$E(a,b,d) = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

Cost Function: Sum of squared distances between each point and the line

with respect to (a,b,d).

1. Minimize E with respect to d:

$$\frac{\partial E}{\partial d} = 0 \Rightarrow d = \frac{1}{n}\sum_{i=1}^{n} ax_i + by_i = a\bar{x} + b\bar{y}$$

Where $(\bar{x}, \bar{y})$ is the mean of the data points

Line Fitting

2. Substitute d back into E

$$E = \sum_{i=1}^{n} [a(x_i - \bar{x}) + b(y_i - \bar{y})]^2 = |\mathcal{U}\mathbf{n}|^2 \quad \text{where} \quad \mathcal{U} = \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \dots & \dots \\ x_n - \bar{x} & y_n - \bar{y} \end{pmatrix}$$

where $\mathbf{n} = (a\ b)^T$.

3. Minimize $E = |Un|^2 = \mathbf{n}^T U^T U \mathbf{n} = \mathbf{n}^T S \mathbf{n}$ with respect to a, b subject to the constraint $\mathbf{n}^T\mathbf{n} = 1$. Note that S is given by

$$S = \mathcal{U}^T\mathcal{U} = \begin{pmatrix} \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 & \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} \\ \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} & \sum_{i=1}^{n} y_i^2 - n\bar{y}^2 \end{pmatrix}$$

which is real, symmetric, and positive definite

Line Fitting

4. This is a constrained optimization problem in $\mathbf{n}$. Solve with Lagrange multiplier

$$L(\mathbf{n}) = \mathbf{n}^T S \mathbf{n} - \lambda(\mathbf{n}^T\mathbf{n} - 1)$$

Take partial derivative (gradient) w.r.t. $\mathbf{n}$ and set to 0.

$$\nabla L = 2S\mathbf{n} - 2\lambda\mathbf{n} = 0$$

or

$$S\mathbf{n} = \lambda\mathbf{n}$$

$\mathbf{n} = (a,b)$ is an Eigenvector of the symmetric matrix S (the one corresponding to the smallest Eigenvalue).

5. d is computed from Step 1.

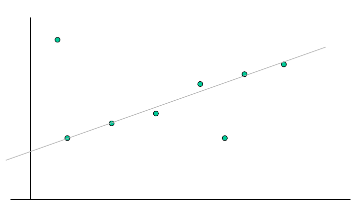
RANSAC

Slides shamelessly taken from

Frank Dellaert and Marc Pollefeys and modified

Simpler Example

- Fitting a straight line



- Inliers
- Outliers

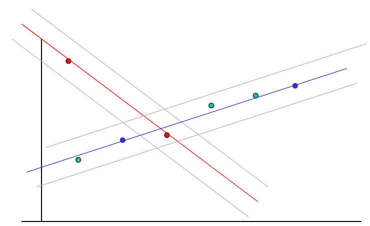
Discard Outliers

- No point with d > t
- RANSAC:
  - RANdom SAmple Consensus
  - Fischler & Bolles 1981
  - Copes with a large proportion of outliers

Main Idea

- Select 2 points at random
- Fit a line
- "Support" = number of inliers
- Line with most inliers wins

Why will this work ?



Best Line has most support

- More support -> better fit

RANSAC

Objective

Robust fit of model to data set S which contains outliers

Algorithm

(i) Randomly select a sample of *s* data points from S and instantiate the model from this subset.

(ii) Determine the set of data points S_i which are within a distance threshold *t* of the model. The set S_i is the **consensus set** of samples and defines the inliers of S.

(iii) If the size of S_i is greater than some threshold *T*, re-estimate the model using all the points in S_i and terminate

(iv) If the size of S_i is less than *T*, select a new subset and repeat the above.

(v) After *N* trials the largest consensus set S_i is selected, and the model is re-estimated using all the points in the subset S_i

Number of trials

Choose *N* (number of trials) so that, with probability *p*, at least one random sample is free from outliers. e.g. *p*=0.99

$$\left(1-(1-e)^s\right)^N = 1-p$$
$$N = \log(1-p)/\log\left(1-(1-e)^s\right)$$

e: proportion of outliers

s: Number of points needed for the model

	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Number of inliers threshold

- Typically, terminate when inlier ratio reaches expected ratio of inliers

$$T = (1-e)N$$

Distance threshold

Choose threshold t so probability for inlier is α (e.g., 0.95)

- Often empirically
- Zero-mean Gaussian noise σ then $d_{\perp}^2$ follows χ_m^2 distribution with m=codimension of model

(codimension=dimension of space – dimension of subspace)

Codimension	Model	t^2
1	E, F, 2D line	$3.84\sigma^2$
2	P	$5.99\sigma^2$

Using RANSAC to estimate the Fundamental Matrix

- What is the model?
- What is the sample size and where do the samples come from?
- What distance do we use to compute the consensus set?
- How often do outliers occur?

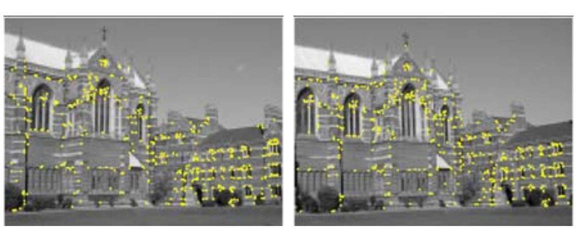
Other models

- 2D motion models
- Typically: points in two images
- Candidates:
  - Translation
  - Euclidean
  - Similarity
  - Affine
  - Projective

Feature Detection and Matching

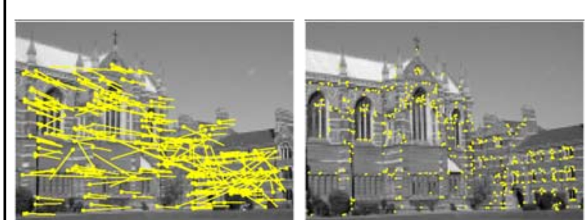
Input Images

Feature Detection and Matching



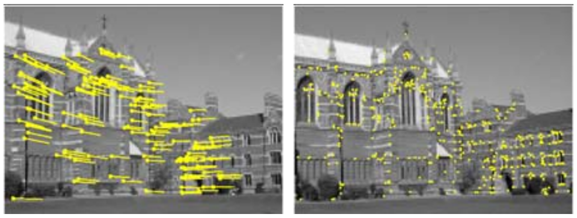
Detected Corners

Feature Detection and Matching



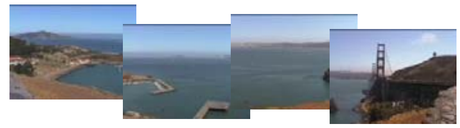
Simple Matching

Feature Detection and Matching



Simple Matching
Including Outlier Rejection

Mosaicing: Homography Estimate with RANSAC



www.cs.cmu.edu/~dellaert/mosaicking

Next Lecture

- Motion
- Reading:
  - Section 10.6.1: Optical Flow and Motion
  - Section 10.6.2: Flow Models
  - Introductory Techniques for 3-D Computer Vision, Trucco and Verri
    - Chapter 8: Motion