Structure from Motion

Introduction to Computer Vision
CSE 152
Lecture 10

Announcements

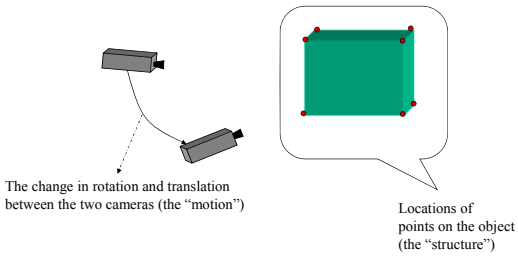
- Graded homework 1 will be returned on Thursday
- Homework 2 is due May 4, 11:59 PM
- Reading:
  - Chapter 8: Structure from Motion
  - Optional: Multiple View Geometry in Computer Vision, 2nd edition, Hartley and Zisserman

Motion

Motion

- Some problems of motion
  - Correspondence: Where have elements of the image moved between image frames?
  - Reconstruction: Given correspondences, what is 3D geometry of scene?
  - Motion segmentation: What are regions of image corresponding to different moving objects?
  - Tracking: Where have objects moved in the image? (related to correspondence and segmentation)

Structure from Motion



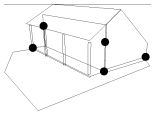
MOVING CAMERAS ARE LIKE STEREO

Structure from Motion (SFM)

- Objective
  - Given two or more images (or video frames), without knowledge of the camera poses (rotations and translations), estimate the camera poses and 3D structure of scene.
- Considerations
  - Discrete motion (wide baseline) vs. continuous (infinitesimal) motion
  - Calibrated vs. uncalibrated
  - Two views vs. multiple views
  - Orthographic (affine) vs. perspective

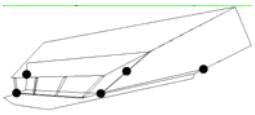
Discrete Motion, Calibrated

- Consider m images of n points, how many unknowns?
  - Unknowns
    - 3D Structure: $3n$
    - First normalized camera $\hat{P} = [I \mid 0]$
      - Rotations: $3(m-1)$
      - Translations (to scale): $3(m-1)-1$
    - Total: $3n + 6(m-1) - 1$
  - Measurements
    - $2nm$
- Solution when $3n + 6(m-1) - 1 \leq 2nm$



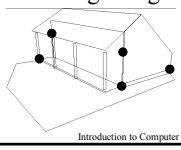
Discrete Motion, Uncalibrated

- Consider m images of n points, how many unknowns?
  - Unknowns
    - 3D Structure: $3n$
    - Cameras (to 3D projective transformation): $11m - 15$
    - Total: $3n + 11m - 15$
  - Measurements
    - $2nm$
- Solution when $3n + 11m - 15 \leq 2nm$



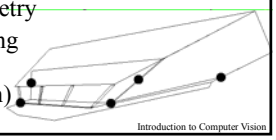
Two Views, Calibrated

- Input: Two images (or video frames)
- Detect feature points
- Determine feature correspondences
- Compute the essential matrix
- Retrieve the relative camera rotation and translation (to scale) from the essential matrix
- Optional: Perform dense stereo matching using recovered epipolar geometry
- Reconstruct corresponding 3D scene points (to scale)



Two Views, Uncalibrated

- Input: Two images (or video frames)
- Detect feature points
- Determine feature correspondences
- Compute the fundamental matrix
- Retrieve the relative camera 3D projective transformation from the fundamental matrix
- Optional: Perform dense stereo matching using recovered epipolar geometry
- Reconstruct corresponding 3D scene points (to 3D projective transformation)



Essential Matrix (Calibrated)

- Number of point correspondences and solutions
  - 5 point correspondences, up to 10 (real) solutions
  - 6 point correspondences, 1 solution
  - 7 point correspondences, 1 or 3 real solutions (and 2 or 0 complex ones)
  - 8 or more point correspondences, 1 solution

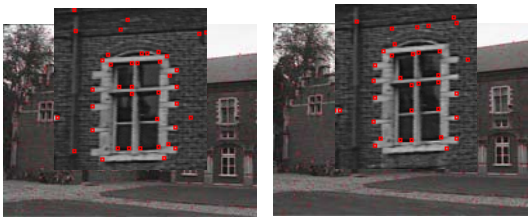
Fundamental Matrix (Uncalibrated)

- Number of point correspondences and solutions
  - 7 point correspondences, 1 or 3 real solutions (and 2 or 0 complex ones)
  - 8 or more point correspondences, 1 solution

Estimation using SVD

- Fundamental matrix
  - Linear estimation (8 or more correspondences)
- Essential matrix
  - Linear estimation (8 or more correspondences)

Feature detection

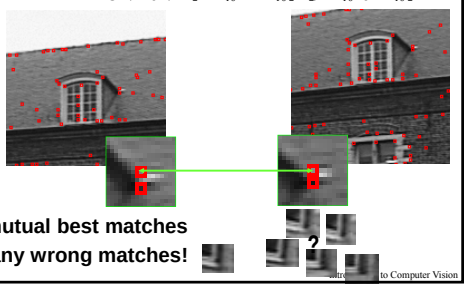


Select strongest features **(e.g. 1000/image)**

Feature matching

Evaluate normalized cross correlation (or sum of squared differences) for all features with similar coordinates

$$\text{e.g.}\ (x', y') \in \left[x - \tfrac{w}{10}, x + \tfrac{w}{10}\right] \times \left[y - \tfrac{h}{10}, y + \tfrac{h}{10}\right]$$



Keep mutual best matches

Still many wrong matches!

Comments

- Greedy Algorithm:
  - Given feature in one image, find best match in second image irrespective of other matches
  - Suitable for small motions, little rotation, small search window
- Otherwise
  - Must compare descriptor over rotation
  - Cannot consider all potential pairings (way too many), so
    - Manual correspondence (e.g., photogrammetry)
    - Use robust outlier rejection (e.g., RANSAC)
    - More descriptive features (line segments, SIFT, larger regions, color)
    - Use video sequence to track, but perform SFM w/ first and last image

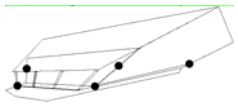
N-View Geometry

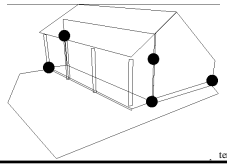
- Reconstruction
  - Bundle adjustment
    - Simultaneous adjustment of parameters for all cameras and all 3D scene points
    - Minimize reprojection error in all images

      $\min_{\hat{\mathbf{P}}^i, \hat{\mathbf{X}}_j} \sum_{ij} d(\hat{\mathbf{P}}^i \hat{\mathbf{X}}_j, \mathbf{x}_j^i)^2$
    - Reconstruction of cameras and 3D scene points to similarity (calibrated) or projective (uncalibrated) ambiguity
  - Factorization (see text)

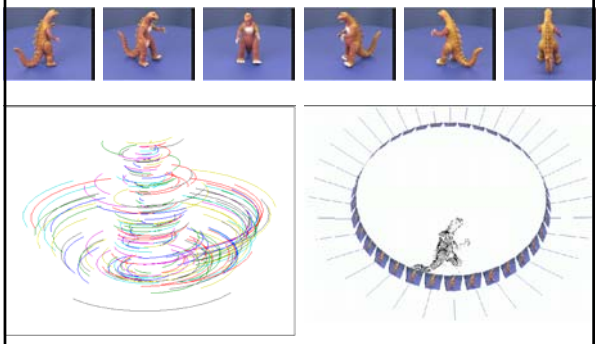
Direct Reconstruction Projective to Euclidean

5 (or more) points correspondences

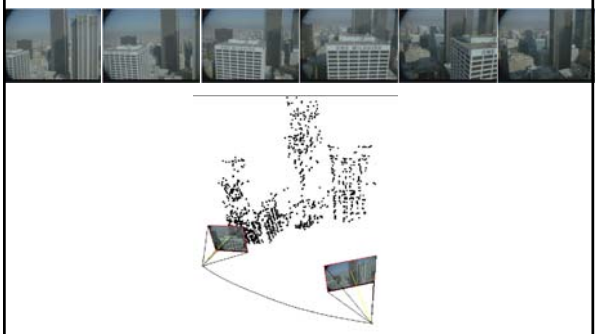




N-View Geometry

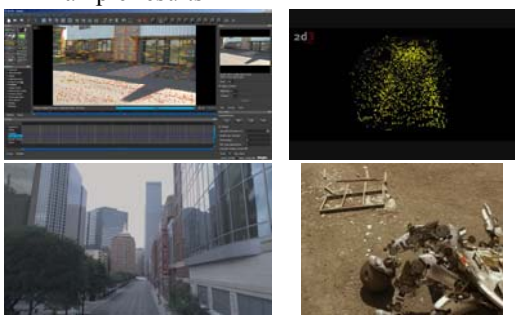


N-View Geometry



N-View Geometry

- Example results



Next Lecture

- Mid-level vision
  - Grouping and model fitting
- Reading:
  - Chapter 10: Grouping and Model Fitting