

## Program Representations

## Representing programs

- Goals

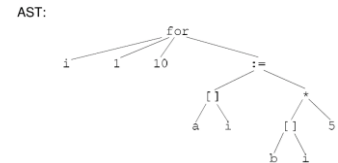
## Representing programs

- Primary goals
  - analysis is easy and effective
    - just a few cases to handle
    - directly link related things
  - transformations are easy to perform
  - general, across input languages and target machines
- Additional goals
  - compact in memory
  - easy to translate to and from
  - tracks info from source through to binary, for source-level debugging, profiling, typed binaries
  - extensible (new opts, targets, language features)
  - displayable

## Option 1: high-level syntax based IR

- Represent source-level structures and expressions directly
- Example: Abstract Syntax Tree

Source:  
 for i := 1 to 10 do  
   a[i] := b[i] \* 5;  
 end



## Option 2: low-level IR

- Translate input programs into low-level primitive chunks, often close to the target machine
- Examples: assembly code, virtual machine code (e.g. stack machines), three-address code, register-transfer language (RTL)

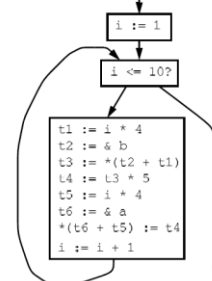
- Standard RTL instrs:

assignment	x := y;
unary op	x := op y;
binary op	x := y op z;
address-of	p := &y;
load	x := *(p + o);
store	*(p + o) := x;
call	x := f(...);
unary compare	op x ?
binary compare	x op y ?

## Option 2: low-level IR

Source:  
 for i := 1 to 10 do  
   a[i] := b[i] \* 5;  
 end

Control flow graph containing RTL instructions:



## Comparison

## Comparison

- Advantages of high-level rep
  - analysis can exploit high-level knowledge of constructs
  - easy to map to source code (debugging, profiling)
- Advantages of low-level rep
  - can do low-level, machine specific reasoning
  - can be language-independent
- Can mix multiple reps in the same compiler

## Components of representation

- Control dependencies: sequencing of operations
  - evaluation of if & then
  - side-effects of statements occur in right order
- Data dependencies: flow of definitions from defs to uses
  - operands computed before operations
- Ideal: represent just dependencies that matter
  - dependencies constrain transformations
  - fewest dependences  $\Rightarrow$  flexibility in implementation



## Control dependencies

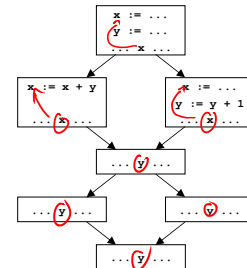
- Option 1: high-level representation
  - control implicit in semantics of AST nodes
- Option 2: control flow graph (CFG)
  - nodes are individual instructions
  - edges represent control flow between instructions
- Options 2b: CFG with basic blocks
  - basic block: sequence of instructions that don't have any branches, and that have a single entry point
  - BB can make analysis more efficient: compute flow functions for an entire BB before start of analysis

## Control dependencies

- CFG does not capture loops very well
- Some fancier options include:
  - the Control Dependence Graph
  - the Program Dependence Graph
- More on this later. Let's first look at data dependencies

## Data dependencies

- Simplest way to represent data dependencies: def/use chains

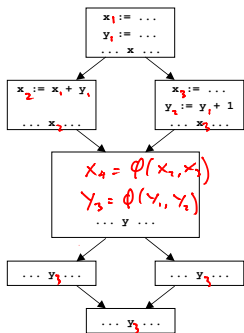


## Def/use chains

- Directly captures dataflow
  - works well for things like constant prop
- But...
- Ignores control flow
  - misses some opt opportunities since conservatively considers all paths
  - not executable by itself (for example, need to keep CFG around)
  - not appropriate for code motion transformations
- Must update after each transformation
- Space consuming

## SSA

- Static Single Assignment
  - invariant: each use of a variable has only one def

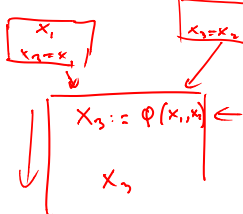


## SSA

- Create a new variable for each def
- Insert  $\phi$  pseudo-assignments at merge points
- Adjust uses to refer to appropriate new names
- Question: how can one figure out where to insert  $\phi$  nodes using a liveness analysis and a reaching defs analysis.

## Converting back from SSA

- Semantics of  $x_3 := \phi(x_1, x_2)$ 
  - set  $x_3$  to  $x_i$  if execution came from  $i$ th predecessor
- How to implement  $\phi$  nodes?



## Converting back from SSA

- Semantics of  $x_3 := \phi(x_1, x_2)$ 
  - set  $x_3$  to  $x_i$  if execution came from  $i$ th predecessor
- How to implement  $\phi$  nodes?
  - Insert assignment  $x_3 := x_1$  along 1<sup>st</sup> predecessor
  - Insert assignment  $x_3 := x_2$  along 2<sup>nd</sup> predecessor
- If register allocator assigns  $x_1$ ,  $x_2$  and  $x_3$  to the same register, these moves can be removed
  - $x_1 \dots x_n$  usually have non-overlapping lifetimes, so this kind of register assignment is legal

## Recall: Common Sub-expression Elim

- Want to compute when an expression is available in a var

- Domain:  $\{x \rightarrow E_1, y \rightarrow E_2, z \rightarrow E_3\}$

$$S = \{x \rightarrow E \mid x \in \text{Var}, E \in \text{Expr}\}$$

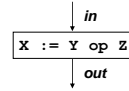
$$P = S$$

$$I = S$$

$$T = \emptyset$$

$$U = \Lambda$$

## Recall: CSE Flow functions

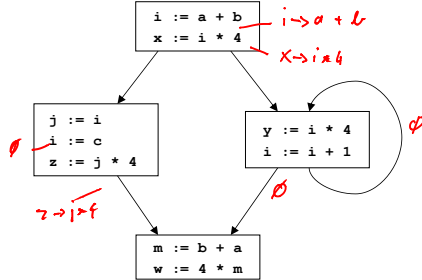


$$F_{X := Y \text{ op } Z}(in) = in - \{X \rightarrow *\} \cup \{X \rightarrow Y \text{ op } Z \mid X \neq Y \wedge X \neq Z\}$$

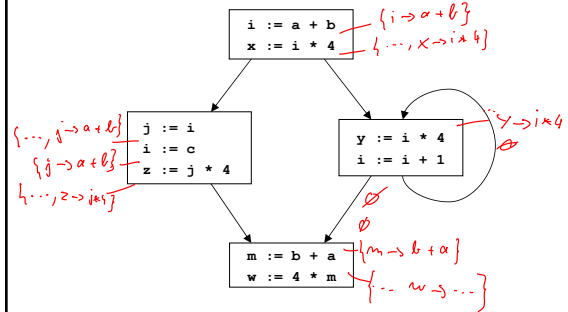


$$F_{X := Y}(in) = in - \{X \rightarrow *\} \cup \{X \rightarrow E \mid Y \rightarrow E \in in\}$$

## Example



## Example



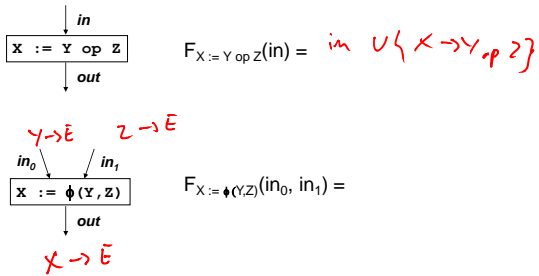
## Problems

- $z := j * 4$  is not optimized to  $z := x$ , even though  $x$  contains the value  $j * 4$
- $m := b + a$  is not optimized, even though  $a + b$  was already computed
- $w := 4 * m$  is not optimized to  $w := x$ , even though  $x$  contains the value  $4 * m$

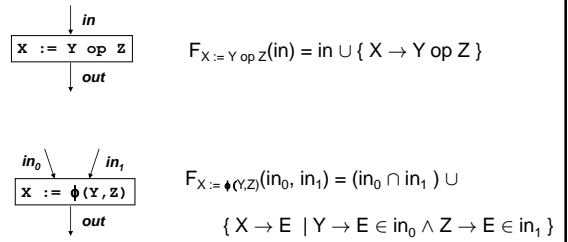
## Problems: more abstractly

- Available expressions overly sensitive to name choices, operand orderings, renamings, assignments
- Use SSA: distinct values have distinct names
- Do copy prop before running available exprs
- Adopt canonical form for commutative ops

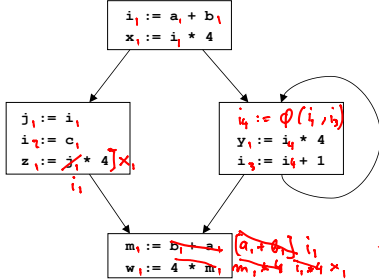
### Example in SSA



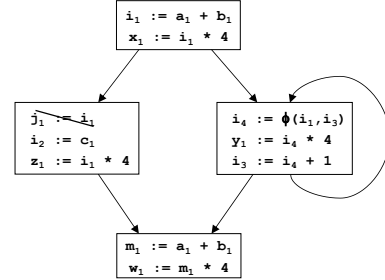
### Example in SSA



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### Example in SSA



### What about pointers?

- Pointers complicate SSA. Several options.
- Option 1: don't use SSA for pointed to variables
- Option 2: adapt SSA to account for pointers
- Option 3: define src language so that variables cannot be pointed to (eg: Java)

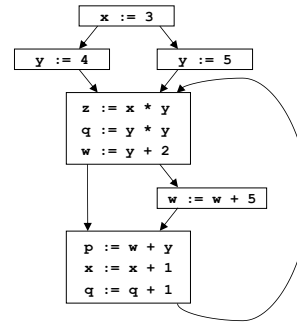
### SSA helps us with CSE

- Let's see what else SSA can help us with
- Loop-invariant code motion

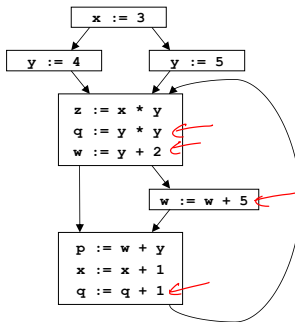
## Loop-invariant code motion

- Two steps: analysis and transformations
- Step 1: find invariant computations in loop
  - invariant: computes same result each time evaluated
- Step 2: move them outside loop
  - to top if used within loop: **code hoisting**
  - to bottom if used after loop: **code sinking**

## Example



## Example



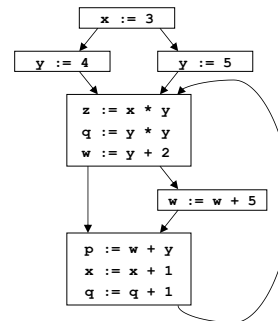
## Detecting loop invariants

- An expression is invariant in a loop L iff:
  - (base cases)
    - it's a constant
    - it's a variable use, **all of whose defs are outside of L**
  - (inductive cases)
    - it's a pure computation all of whose args are loop-invariant
    - it's a variable use with **only one reaching def**, and the rhs of that def is loop-invariant

## Computing loop invariants

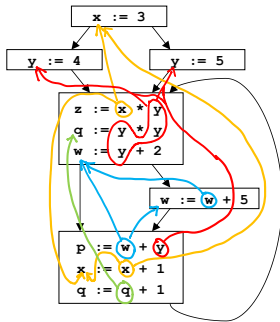
- Option 1: iterative dataflow analysis
  - optimistically assume all expressions loop-invariant, and propagate
- Option 2: build def/use chains
  - follow chains to identify and propagate invariant expressions
- Option 3: SSA
  - like option 2, but using SSA instead of def/use chains

## Example using def/use chains



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## Example using def/use chains

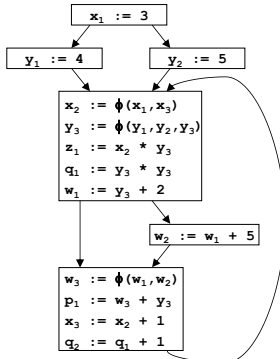


- An expression is invariant in a loop L iff:
  - (base cases)
    - it's a constant
    - it's a variable use, all of whose **single** defs are outside of L
  - (inductive cases)
    - it's a pure computation all of whose args are loop-invariant
    - it's a variable use with **only one** reaching def, and the rhs of that def is loop-invariant

## Loop invariant detection using SSA

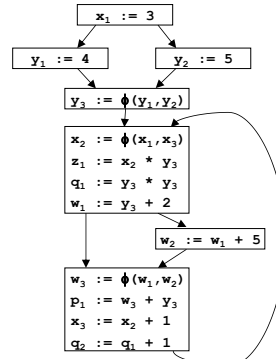
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- $\phi$  functions are not pure

## Example using SSA



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- $\phi$  functions are not pure

## Example using SSA and preheader



- An expression is invariant in a loop L iff:
  - (base cases)
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  - (inductive cases)
    - it's a pure computation all of whose args are loop-invariant
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- $\phi$  functions are not pure

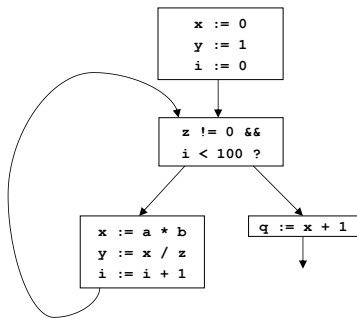
## Summary: Loop-invariant code motion

- Two steps: analysis and transformations
- Step 1: find invariant computations in loop
  - invariant: computes same result each time evaluated
- Step 2: move them outside loop
  - to top if used within loop: **code hoisting**
  - to bottom if used after loop: **code sinking**

## Code motion

- Say we found an invariant computation, and we want to move it out of the loop (to loop pre-header)
- When is it legal?
- Need to preserve relative order of invariant computations to preserve data flow among move statements
- Need to preserve relative order between invariant computations and other computations

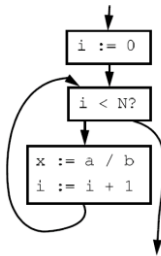
## Example



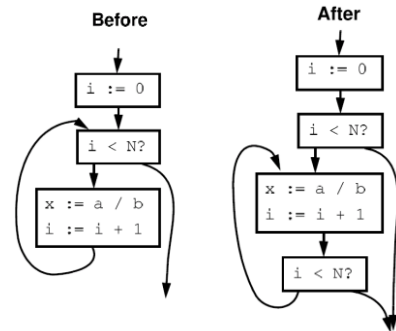
## Lesson from example: domination restriction

- To move statement S to loop pre-header, S must **dominate** all loop exits  
[ A dominates B when all paths to B first pass through A ]
- Otherwise may execute S when never executed otherwise
- If S is pure, then can relax this constraint at cost of possibly slowing down the program

## Domination restriction in for loops



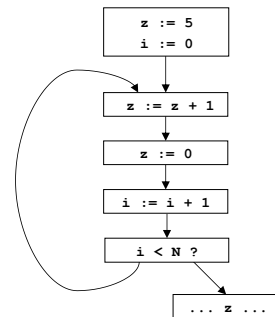
## Domination restriction in for loops



## Avoiding domination restriction

- Domination restriction strict
  - Nothing inside branch can be moved
  - Nothing after a loop exit can be moved
- Can be circumvented through loop normalization
  - while-do => if-do-while

## Another example

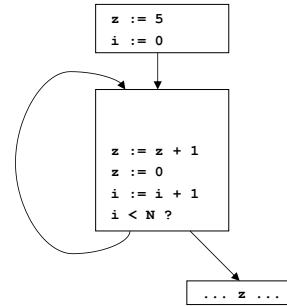




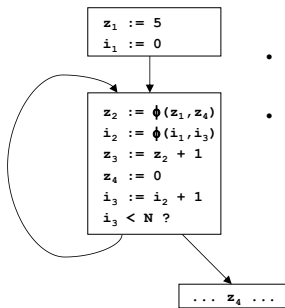
## Data dependence restriction

- To move S:  $z := x \text{ op } y$ :  
S must be the only assignment to  $z$  in loop, and no use of  $z$  in loop reached by any def other than S
- Otherwise may reorder defs/uses

## Avoiding data restriction



## Avoiding data restriction



- Restriction unnecessary in SSA!!!
- Implementation of phi nodes as moves will cope with re-ordered defs/uses

## Summary of Data dependencies

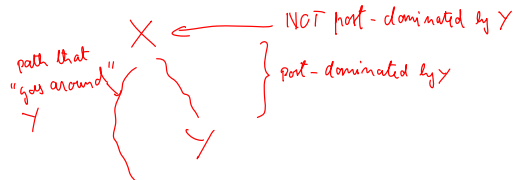
- We've seen SSA, a way to encode data dependencies better than just def/use chains
  - makes CSE easier
  - makes loop invariant detection easier
  - makes code motion easier
- Now we move on to looking at how to encode control dependencies

## Control Dependencies

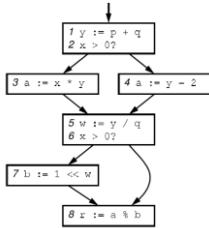
- A node (basic block) Y is control-dependent on another X iff X determines whether Y executes
  - there exists a path from X to Y s.t. every node in the path other than X and Y is post-dominated by Y
  - X is not post-dominated by Y

## Control Dependencies

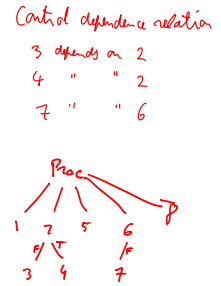
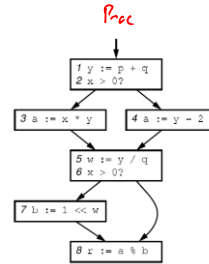
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### Example



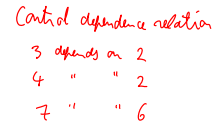
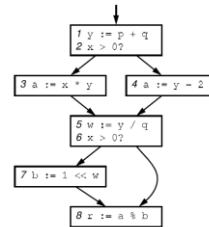
### Example



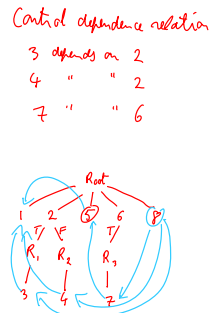
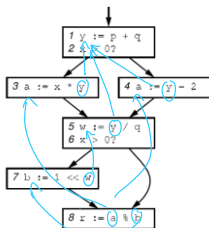
### Control Dependence Graph

- Control dependence graph: Y descendent of X iff Y is control dependent on X
  - label each child edge with required condition
  - group all children with same condition under region node
- Program dependence graph: super-impose dataflow graph (in SSA form or not) on top of the control dependence graph

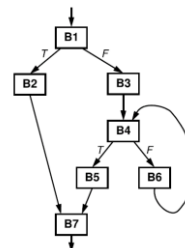
### Example



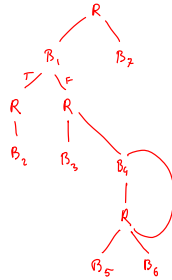
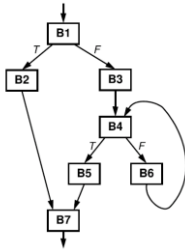
### Example



### Another example



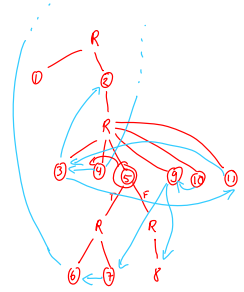
## Another example



## Another example

```

1 i1 := 0;
while 2 do
3 i3 := phi(i1, i2);
4 x := i3 * b;
5 if 6 then
6 w := c * c;
7 y1 := 9 + w;
8 else
9 y2 := 9;
10 y3 := phi(y1, y2);
11 print(y3);
12 i2 := i3 + 1;
end
  
```



## Summary of Control Dependence Graph

- More flexible way of representing control-dependencies than CFG (less constraining)
- Makes code motion a local transformation
- However, much harder to convert back to an executable form

## Course summary so far

- Dataflow analysis
  - flow functions, lattice theoretic framework, optimistic iterative analysis, precision, MOP
- Advanced Program Representations
  - SSA, CDG, PDG
- Along the way, several analyses and opts
  - reaching defs, const prop & folding, available exprs & CSE, liveness & DAE, loop invariant code motion
- Pointer analysis
  - Andersen, Steensgaard, and long the way: flow-insensitive analysis
- Next: dealing with procedures