Another example: constant prop

- Set D =

in
`X := N`
out

$F_{X := N}(in) =$

in
`X := Y op Z`
out

$F_{X := Y \text{ op } Z}(in) =$

Another example: constant prop

- Set D = $2^{\{ x \to N \mid x \in Vars \wedge N \in Z \}}$

in
`X := N`
out

$F_{X := N}(in) = in - \{ X \to * \} \cup \{ X \to N \}$

in
`X := Y op Z`
out

$F_{X := Y \text{ op } Z}(in) = in - \{ X \to * \} \cup$
$\{ X \to N \mid (Y \to N_1) \in in \wedge$
$(Z \to N_2) \in in \wedge$
$N = N_1 \text{ op } N_2 \}$

Another example: constant prop

in
`X := *Y`
out

$F_{X := *Y}(in) =$

in
`*X := Y`
out

$F_{*X := Y}(in) =$

Another example: constant prop

in
`X := *Y`
out

$F_{X := *Y}(in) = in - \{ X \to * \}$
$\cup \{ X \to N \mid \forall Z \in \text{may-point-to}(Y) .$
$(Z \to N) \in in \}$

in
`*X := Y`
out

$F_{*X := Y}(in) = in - \{ Z \to * \mid Z \in \text{may-point}(X) \}$
$\cup \{ Z \to N \mid Z \in \text{must-point-to}(X) \wedge$
$Y \to N \in in \}$
$\cup \{ Z \to N \mid (Y \to N) \in in \wedge$
$(Z \to N) \in in \}$

Another example: constant prop

in
`*X := *Y + *Z`
out

$F_{*X := *Y + *Z}(in) =$

in
`X := G(...)`
out

$F_{X := G(...)}(in) =$

Another example: constant prop

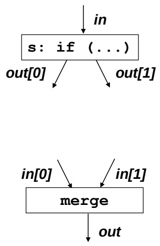
in
`*X := *Y + *Z`
out

$F_{*X := *Y + *Z}(in) = F_{a := *Y; b := *Z; c := a + b; *X := c}(in$

in
`X := G(...)`
out

$F_{X := G(...)}(in) = \emptyset$

Another example: constant prop



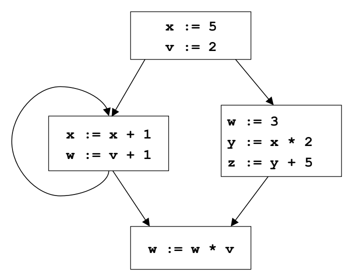
Lattice

- (D, ⊑, ⊥, ⊤, ⊔, ⊓) =

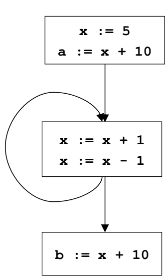
Lattice

- (D, ⊑, ⊥, ⊤, ⊔, ⊓) =
  (2^A, ⊇, A, ∅, ∩, ∪)
  where A = { x → N | x ∈ Vars ∧ N ∈ Z }

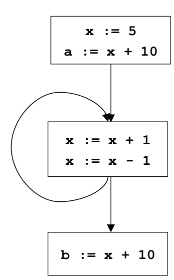
Example



Another Example



Another Example starting at top



Back to lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^{A}, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \rightarrow N \mid x \in \text{Vars} \wedge N \in Z \}$

- What's the problem with this lattice?

Back to lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^{A}, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \rightarrow N \mid x \in \text{Vars} \wedge N \in Z \}$

- What's the problem with this lattice?

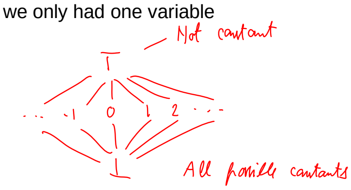
- Lattice is infinitely high, which means we can't guarantee termination

Better lattice

- Suppose we only had one variable

Better lattice

- Suppose we only had one variable



- $D = \{\bot, \top\} \cup Z$
- $\forall i \in Z \,.\, \bot \sqsubseteq i \wedge i \sqsubseteq \top$
- height = 3

For all variables

- Two possibilities
- Option 1: Tuple of lattices
- Given lattices $(D_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1)$ ... $(D_n, \sqsubseteq_n, \bot_n, \top_n, \sqcup_n, \sqcap_n)$ create:

  tuple lattice D^n =

For all variables

- Two possibilities
- Option 1: Tuple of lattices
- Given lattices $(D_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1)$ ... $(D_n, \sqsubseteq_n, \bot_n, \top_n, \sqcup_n, \sqcap_n)$ create:

  tuple lattice $D^n = ((D_1 \times \ldots \times D_n), \sqsubseteq, \bot, \top, \sqcup, \sqcap)$ where
  $\bot = (\bot_1, \ldots, \bot_n)$
  $\top = (\top_1, \ldots, \top_n)$
  $(a_1, \ldots, a_n) \sqcup (b_1, \ldots, b_n) = (a_1 \sqcup_1 b_1, \ldots, a_n \sqcup_n b_n)$
  $(a_1, \ldots, a_n) \sqcap (b_1, \ldots, b_n) = (a_1 \sqcap_1 b_1, \ldots, a_n \sqcap_n b_n)$
  height = height(D_1) + ... + height(D_n)

For all variables

- Option 2: Map from variables to single lattice
- Given lattice (D, $\sqsubseteq_1$, $\bot_1$, $\top_1$, $\sqcup_1$, $\sqcap_1$) and a set V, create:

map lattice $V \to D = (V \to D, \sqsubseteq, \bot, \top, \sqcup, \sqcap)$

$\bot = \lambda v \to \bot_1$

Back to example

in

`X := Y op Z`

out

$F_{X := Y \text{ op } Z}(\text{in}) =$

Back to example

in

`X := Y op Z`

out

$F_{X := Y \text{ op } Z}(\text{in}) = \text{in} [\ X \to \text{in}(Y)\ \widehat{\text{op}}\ \text{in}(Z)\ ]$

where a $\widehat{\text{op}}$ b =

$\widehat{op}$	$\bot$	d_1	$\top$
$\bot$	$\bot$	$\bot$	$\top$
d_2	$\bot$	$d_1\ op\ d_2$	$\top$
$\top$	$\top$	$\top$	$\top$

General approach to domain design

- Simple lattices:
  - boolean logic lattice
  - powerset lattice
  - incomparable set: set of incomparable values, plus top and bottom (eg const prop lattice)
  - two point lattice: just top and bottom
- Use combinators to create more complicated lattices
  - tuple lattice constructor
  - map lattice constructor

May vs Must

- Has to do with definition of computed info
- Set of $x \to y$ must-point-to pairs
  - if we compute $x \to y$, then, then during program execution, x must point to y
- Set of $x \to y$ may-point-to pairs
  - if during program execution, it is possible for x to point to y, then we must compute $x \to y$

May vs must

	May	Must
most optimistic (bottom)		
most conservative (top)		
safe		
merge		

May vs must

	May	Must
most optimistic (bottom)	empty set	full set
most conservative (top)	full set	empty set
safe	overly big	overly small
merge	∪	∩

Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain:

Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain:

$S = \{ X \to E \mid X \in Var, E \in Expr \}$

$D = 2^S$

$\bot = S$

$\top = \emptyset$

$\sqcup = \cap$

Flow functions

in

```
X := Y op Z
```

out

$F_{X := Y \text{ op } Z}(\text{in}) =$

in

```
X := Y
```

out

$F_{X := Y}(\text{in}) =$

Flow functions

in

```
X := Y op Z
```

out

$$F_{X := Y \text{ op } Z}(\text{in}) = \text{in} - \{ X \to * \} - \{ * \to \ldots X \ldots \} \cup \{ X \to Y \text{ op } Z \mid X \neq Y \land X \neq Z \}$$

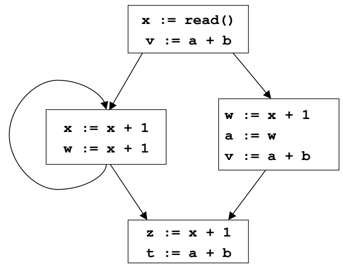
in

```
X := Y
```

out

$$F_{X := Y}(\text{in}) = \text{in} - \{ X \to * \} - \{ * \to \ldots X \ldots \} \cup \{ X \to E \mid Y \to E \in \text{in} \}$$

Example



Direction of analysis

- Although constraints are not directional, flow functions are
- All flow functions we have seen so far are in the forward direction
- In some cases, the constraints are of the form
  in = F(out)
- These are called backward problems.
- Example: live variables
  - compute the set of variables that may be live

Live Variables

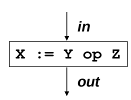
- A variable is live at a program point if it will be used before being redefined
- A variable is dead at a program point if it is redefined before being used

Example: live variables

- Set D =
- Lattice: $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap)$ =

Example: live variables

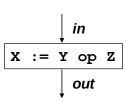
- Set D = 2^{Vars}
- Lattice: $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) = (2^{\text{Vars}}, \subseteq, \emptyset, \text{Vars}, \cup, \cap)$



$F_{X := Y \text{ op } Z}(\text{out})$ =

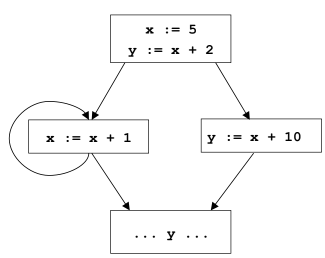
Example: live variables

- Set D = 2^{Vars}
- Lattice: $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) = (2^{\text{Vars}}, \subseteq, \emptyset, \text{Vars}, \cup, \cap)$

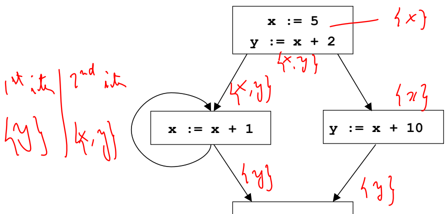


$F_{X := Y \text{ op } Z}(\text{out}) = \text{out} - \{ X \} \cup \{ Y, Z\}$

Example: live variables

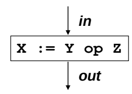


Example: live variables



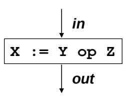
How can we remove the `x := x + 1` stmt?

Revisiting assignment



$F_{X := Y \text{ op } Z}(\text{out}) = \text{out} - \{ X \} \cup \{ Y, Z\}$

Revisiting assignment



$F_{X := Y \text{ op } Z}(\text{out}) = \text{out} - \{ X \} \cup \{ Y, Z\}$

out - {x} ∪

x ∉ out ? ∅ : {Y, Z}

Theory of backward analyses

- Can formalize backward analyses in two ways
- Option 1: reverse flow graph, and then run forward problem
- Option 2: re-develop the theory, but in the backward direction

Precision

- Going back to constant prop, in what cases would we lose precision?

Precision

- Going back to constant prop, in what cases would we lose precision?

```
x := 5
if (<expr>) {
   x := 6
}
... x ...

where <expr> is
equiv to false
```

```
if (p) {
   x := 5;
} else
   x := 4;
}
...
if (p) {
   y := x + 1
} else {
   y := x + 2
}
... y ...
```

```
if (...) {
   x := -1;
} else
   x := 1;
}
y := x * x;
... y ...
```

Precision

- The first problem: Unreachable code
  - solution: run unreachable code removal before
  - the unreachable code removal analysis will do its best, but may not remove all unreachable code
- The other two problems are path-sensitivity issues
  - Branch correlations: some paths are infeasible
  - Path merging: can lead to loss of precision

MOP: meet over all paths

- Information computed at a given point is the meet of the information computed by each path to the program point

```
if (...) {
   x := -1;
} else
   x := 1;
}
y := x * x;
... y ...
```

MOP

- For a path p, which is a sequence of statements $[s_1, ..., s_n]$, define: $F_p(in) = F_{s_n}(...F_{s_1}(in) ...)$
- In other words: $F_p = F_{s_1} \circ \cdots \circ F_{s_n}$
- Given an edge e, let paths-to(e) be the (possibly infinite) set of paths that lead to e
- Given an edge e, MOP(e) = $\bigsqcup_{p \in \text{paths-to}(e)} F_p(\bot)$
- For us, should be called JOP (ie: join, not meet)

MOP vs. dataflow

- MOP is the "best" possible answer, given a fixed set of flow functions
  - This means that MOP $\sqsubseteq$ dataflow at edge in the CFG
- In general, MOP is not computable (because there can be infinitely many paths)
  - vs dataflow which is generally computable (if flow fns are monotonic and height of lattice is finite)
- And we saw in our example, in general, MOP $\neq$ dataflow

MOP vs. dataflow

- However, it would be great if by imposing some restrictions on the flow functions, we could guarantee that dataflow is the same as MOP. What would this restriction be?

Dataflow

```
x := -1;        x := 1;
       \       /
         Merge
      y := x * x;
      ... y ...
```

MOP

```
x := -1;        x := 1;
y := x * x;     y := x * x;
... y ...       ... y ...
       \       /
         Merge
```

MOP vs. dataflow

- However, it would be great if by imposing some restrictions on the flow functions, we could guarantee that dataflow is the same as MOP. What would this restriction be?
- Distributive problems. A problem is distributive if:

  $\forall$ a, b . F(a $\sqcup$ b) = F(a) $\sqcup$ F(b)

- If flow function is distributive, then MOP = dataflow

Summary of precision

- Dataflow is the basic algorithm
- To basic dataflow, we can add path-separation
  - Get MOP, which is same as dataflow for distributive problems
  - Variety of research efforts to get closer to MOP for non-distributive problems
- To basic dataflow, we can add path-pruning
  - Get branch correlation
- To basic dataflow, can add both:
  - meet over all feasible paths