Dataflow analysis

Dataflow analysis: what is it?

- A common framework for expressing algorithms that compute information about a program
- Why is such a framework useful?

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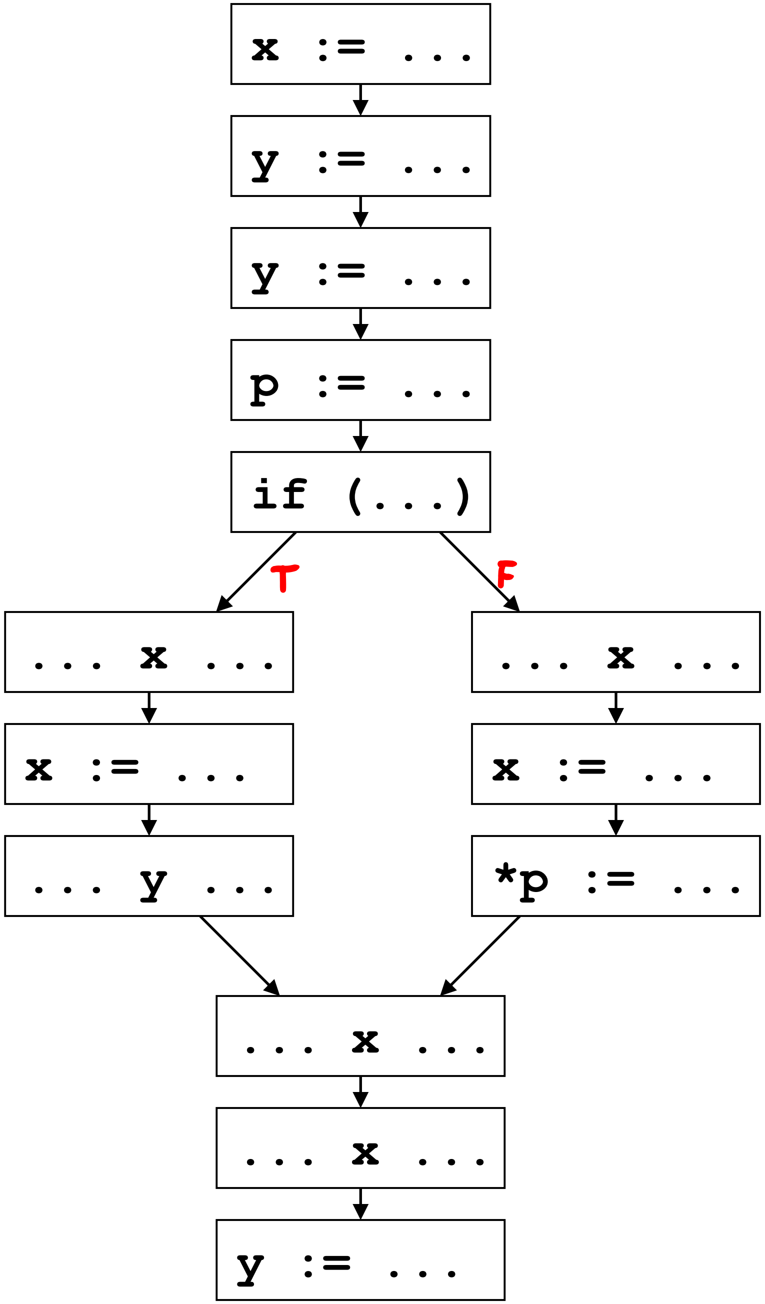
- A common framework for expressing algorithms that compute information about a program
- Why is such a framework useful?
- Provides a common language, which makes it easier to:
  - communicate your analysis to others
  - compare analyses
  - adapt techniques from one analysis to another
  - reuse implementations (eg: dataflow analysis frameworks)

Control Flow Graphs

- For now, we will use a Control Flow Graph representation of programs
  - each statement becomes a node
  - edges between nodes represent control flow
- Later we will see other program representations
  - variations on the CFG (eg CFG with basic blocks)
  - other graph based representations

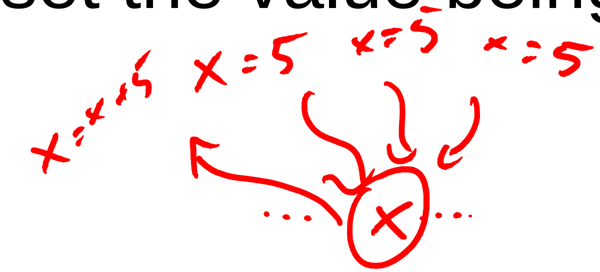
Example CFG

```
x := ...
y := ...
y := ...
p := ...
if (...) {
   ... x ...
   x := ...
   ... y ...
}
else {
   ... x ...
   x := ...
   *p := ...
}
... x ...
... y ...
y := ...
```

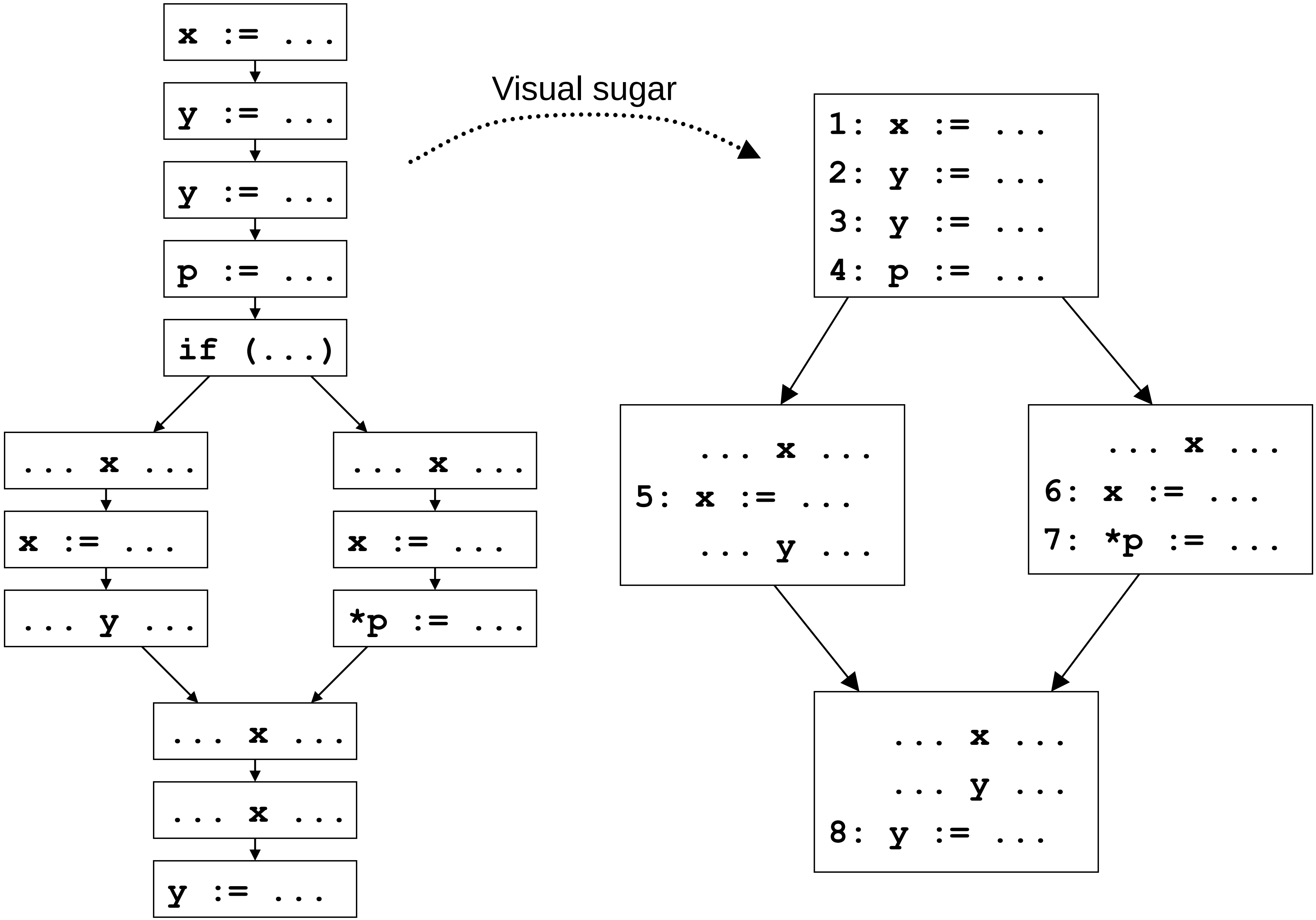


An example DFA: reaching definitions

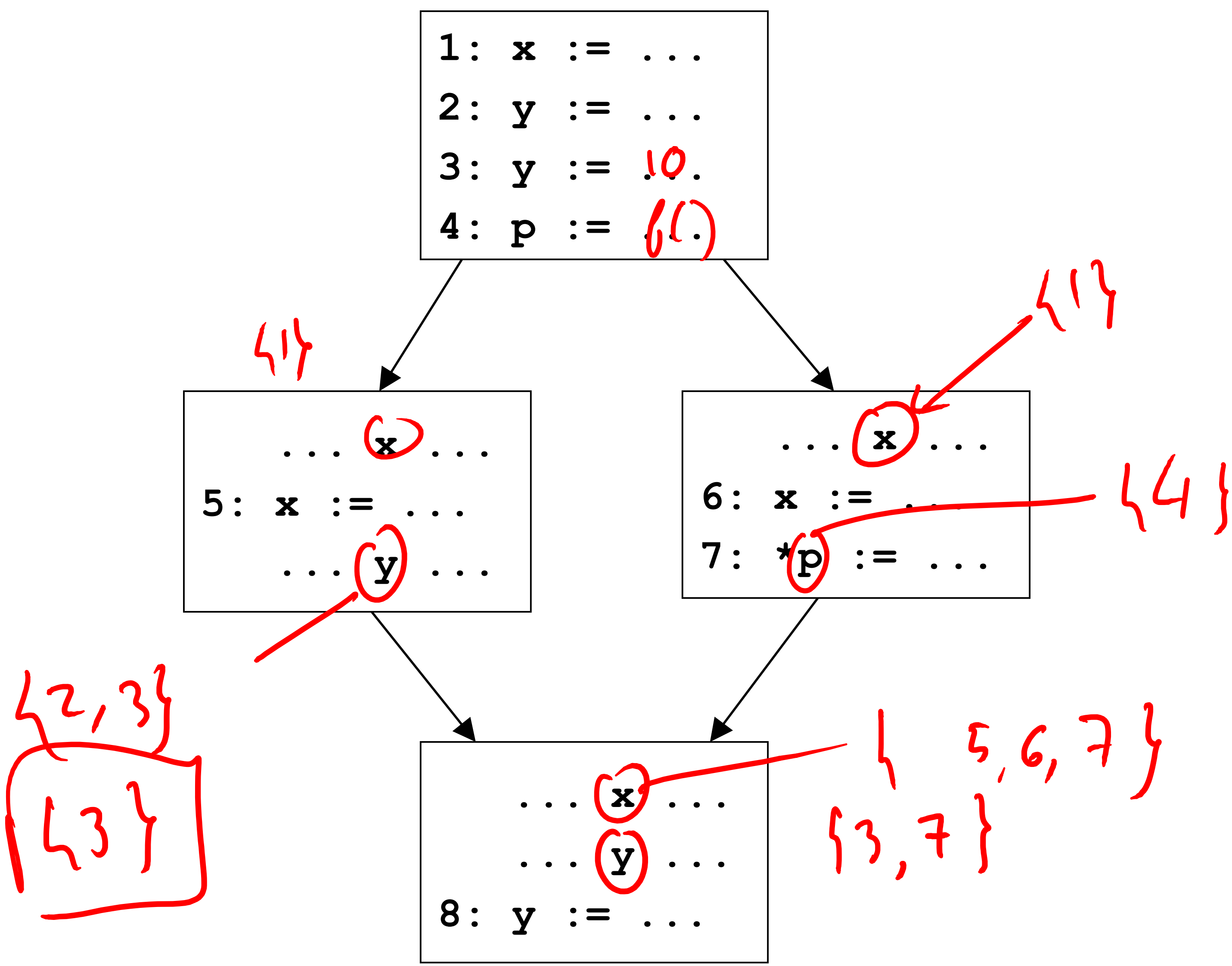
- For each use of a variable, determine what assignments could have set the value being read from the variable
- Information useful for:
  - performing constant and copy prop
  - detecting references to undefined variables
  - presenting "def/use chains" to the programmer
  - building other representations, like the DFG

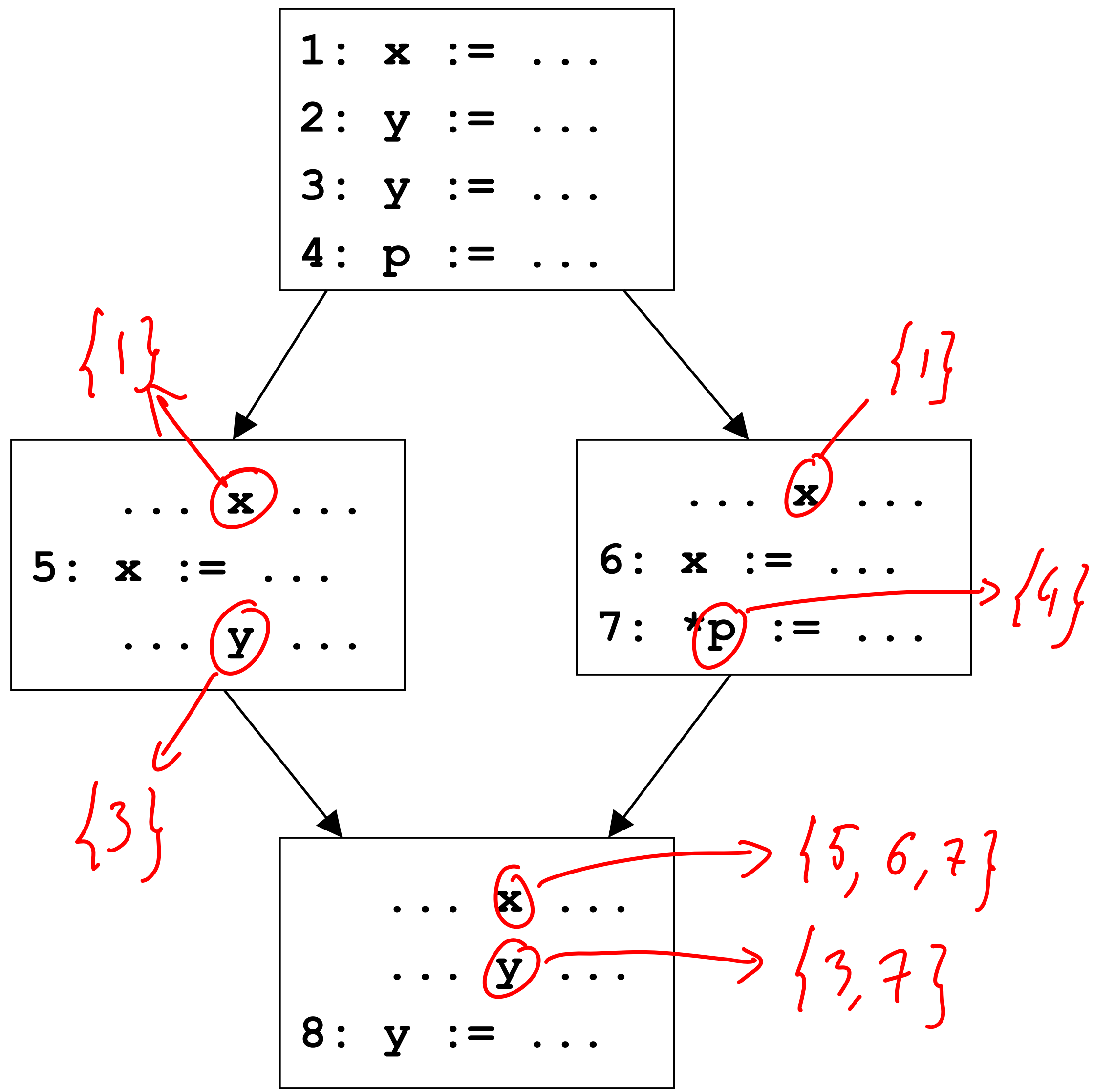


- Let's try this out on an example



x := ...
y := ...
y := ...
p := ...
if (...)
... x ...
x := ...
... y ...
... x ...
x := ...
*p := ...
... x ...
... x ...
y := ...
Visual sugar
1: x := ...
2: y := ...
3: y := ...
4: p := ...
... x ...
5: x := ...
... y ...
... x ...
6: x := ...
7: *p := ...
... x ...
... y ...
8: y := ...


1: x := ...
2: y := ...
3: y := 10
4: p := f()
{1}
... x ...
5: x := ...
... y ...
{1}
... x ...
6: x := ...
7: *p := ...
{4}
{2, 3}
{3}
... x ...
... y ...
8: y := ...
{5, 6, 7}
{3, 7}


1: x := ...
2: y := ...
3: y := ...
4: p := ...
... x ...
5: x := ...
... y ...
... x ...
6: x := ...
7: *p := ...
... x ...
... y ...
8: y := ...
{1}
{3}
{1}
{4}
{5, 6, 7}
{3, 7}

Safety

- When is computed info safe?
- Recall intended use of this info:
  - performing constant and copy prop
  - detecting references to undefined variables
  - presenting “def/use chains” to the programmer
  - building other representations, like the DFG
- Safety:
  - can have more bindings than the “true” answer, but can’t miss any

Reaching definitions generalized

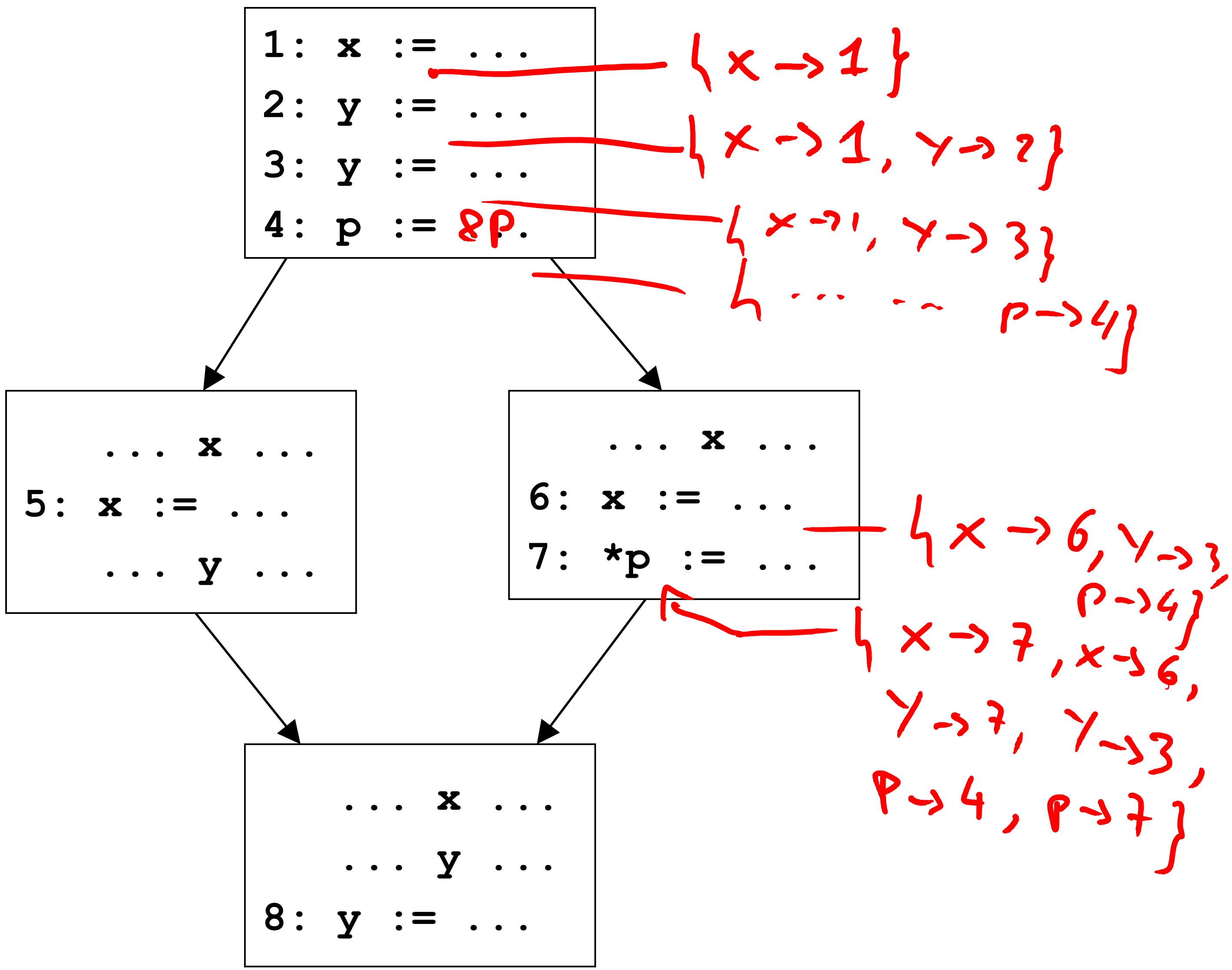
- DFA framework geared to computing information at each program point (edge) in the CFG
  - So generalize problem by stating what should be computed at each program point
- For each program point in the CFG, compute the set of definitions (statements) that may reach that point
- Notion of safety remains the same

Reaching definitions generalized

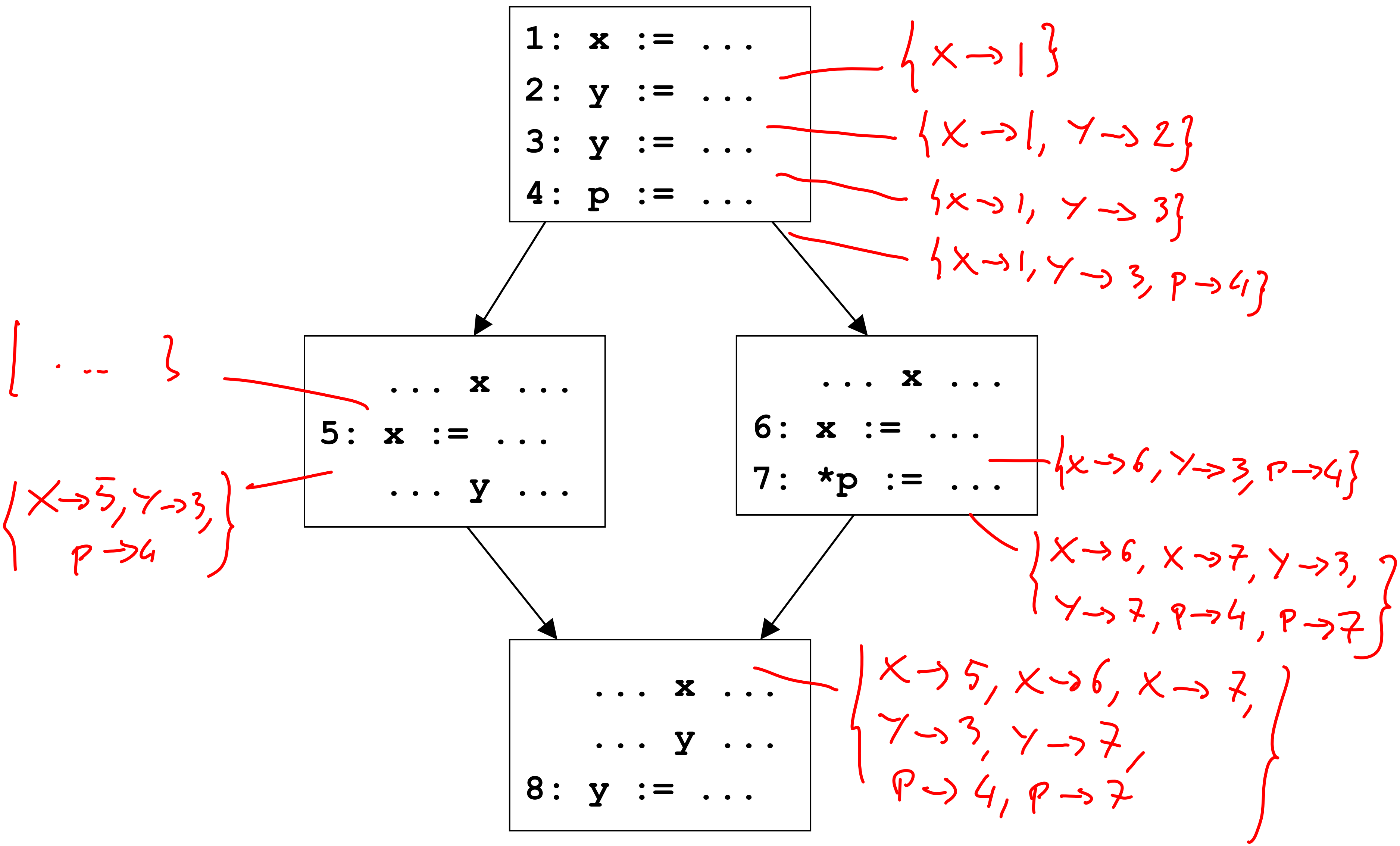
- Computed information at a program point is a set of var $\rightarrow$ stmt bindings
  - eg: $\{ x \rightarrow s_1, x \rightarrow s_2, y \rightarrow s_3 \}$
- How do we get the previous info we wanted?
  - if a var *x* is used in a stmt whose incoming info is *in*, then:

Reaching definitions generalized

- Computed information at a program point is a set of var $\rightarrow$ stmt bindings
  - eg: $\{ x \rightarrow s_1, x \rightarrow s_2, y \rightarrow s_3 \}$
- How do we get the previous info we wanted?
  - if a var *x* is used in a stmt whose incoming info is *in*, then: $\{ s \mid (x \rightarrow s) \in in \}$
- This is a common pattern
  - generalize the problem to define what information should be computed at each program point
  - use the computed information at the program points to get the original info we wanted



1: x := ...
2: y := ...
3: y := ...
4: p := &p
{x -> 1}
{x -> 1, y -> 2}
{x -> 1, y -> 3}
{... ... p -> 4}
... x ...
5: x := ...
... y ...
... x ...
6: x := ...
7: *p := ...
{x -> 6, y -> 3, p -> 4}
{x -> 7, x -> 6, y -> 7, y -> 3, p -> 4, p -> 7}
... x ...
... y ...
8: y := ...



1: x := ...
2: y := ...
3: y := ...
4: p := ...
{x → 1}
{x → 1, y → 2}
{x → 1, y → 3}
{x → 1, y → 3, p → 4}
{ ... }
... x ...
5: x := ...
... y ...
{x → 5, y → 3, p → 4}
... x ...
6: x := ...
7: *p := ...
{x → 6, y → 3, p → 4}
{x → 6, x → 7, y → 3, y → 7, p → 4, p → 7}
... x ...
... y ...
8: y := ...
{x → 5, x → 6, x → 7, y → 3, y → 7, p → 4, p → 7}

Using constraints to formalize DFA

- Now that we’ve gone through some examples, let’s try to precisely express the algorithms for computing dataflow information
- We’ll model DFA as solving a system of constraints
- Each node in the CFG will impose constraints relating information at predecessor and successor points
- Solution to constraints is result of analysis

Constraints for reaching definitions

in

```
S: X := ...
```

out

$$out = in - \{X \to *\} \cup \{X \to S\}$$

in

```
S: *P := ...
```

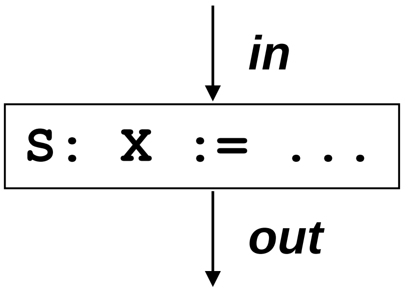
out

$$[may\text{-}pt(P)]$$

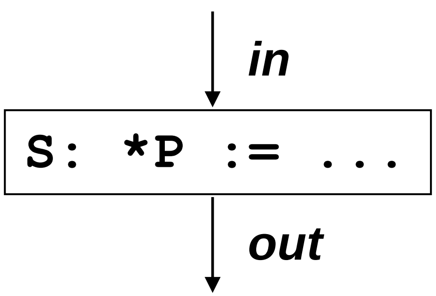
$$\underline{out = in \cup \{* \to S\}}$$

$$in \cup \{X \to S \mid X \in may\text{-}pt(P)\} - \{X \to * \mid X \in must\text{-}pt(P)\}$$

Constraints for reaching definitions



$$out = in - \{ X \rightarrow S' \mid S' \in stmts \} \cup \{ X \rightarrow S \}$$



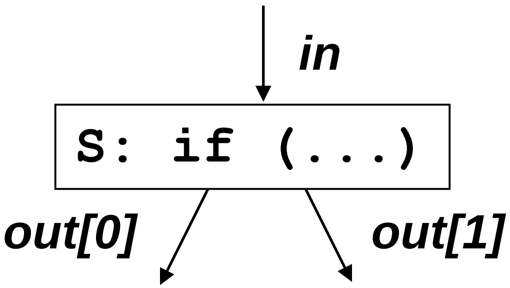
- Using may-point-to information:

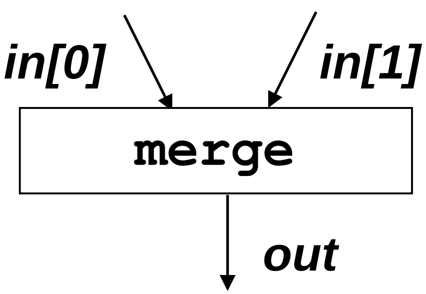
$$out = in \cup \{ X \rightarrow S \mid X \in \text{may-point-to}(P) \}$$

- Using must-point-to aswell:

$$\begin{aligned} out = in - \{ X \rightarrow S' \mid\ & X \in \text{must-point-to}(P) \ \wedge \\ & S' \in stmts \} \\ \cup \{ X \rightarrow S \mid\ & X \in \text{may-point-to}(P) \} \end{aligned}$$

Constraints for reaching definitions

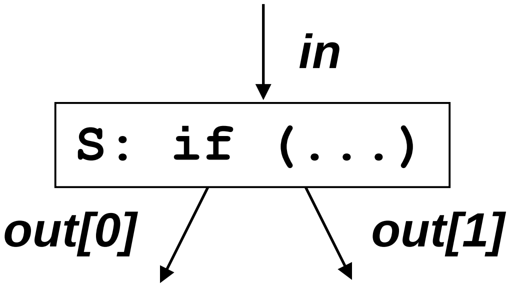




if (*p = 3) {

}

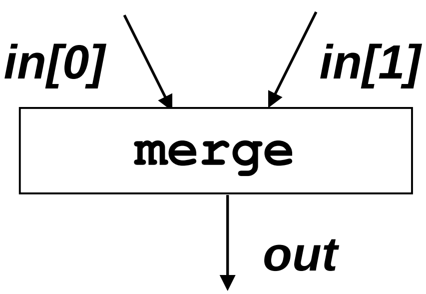
Constraints for reaching definitions



$out\,[\,0\,] = in \wedge$

$out\,[\,1\,] = in$

more generally: $\forall\, i\,.\; out\,[\,i\,] = in$



$out = in\,[\,0\,] \;\cup\; in\,[\,1\,]$

more generally: $out = \bigcup_{i}\, in\,[\,i\,]$

Flow functions

- The constraint for a statement kind s often have the form: out = F_s(in)
- F_s is called a flow function
  - other names for it: dataflow function, transfer function
- Given information *in* before statement s, F_s(*in*) returns information after statement s
- Other formulations have the statement s as an explicit parameter to F: given a statement *s* and some information *in*, F(*s*,*in*) returns the outgoing information after statement s

Flow functions, some issues

- Issue: what does one do when there are multiple input edges to a node?

- Issue: what does one do when there are multiple outgoing edges to a node?

Flow functions, some issues

- Issue: what does one do when there are multiple input edges to a node?
  - the flow functions takes as input a tuple of values, one value for each incoming edge
- Issue: what does one do when there are multiple outgoing edges to a node?
  - the flow function returns a tuple of values, one value for each outgoing edge
  - can also have one flow function per outgoing edge

Flow functions

- Flow functions are a central component of a dataflow analysis
- They state constraints on the information flowing into and out of a statement
- This version of the flow functions is local
  - it applies to a particular statement kind
  - we'll see global flow functions shortly...

Summary of flow functions

- Flow functions: Given information *in* before statement s, $F_s(in)$ returns information after statement s
- Flow functions are a central component of a dataflow analysis
- They state constraints on the information flowing into and out of a statement

Back to example

```
1: x := ...
2: y := ...
3: y := ...
4: p := ...
   if(...)
```

d_0, d_1, d_2, d_3, d_4

$d_1 = F_a(d_0)$

$d_2 = F_b(d_1)$

$d_3 = F_c(d_2)$

$d_4 = F_d(d_3)$

$d_5 = F_e(d_4)$

$d_9 = F_f(d_4)$

d_9 d_5

$d_{10} = F_j(d_9)$

$d_{11} = F_k(d_{10})$

$d_{12} = F_l(d_{11})$

d_{10}, d_{11}

```
   ... x ...
5: x := ...
   ... y ...
```

```
   ... x ...
6: x := ...
7: *p := ...
```

d_6, d_7

$d_6 = F_g(d_5)$

$d_7 = F_h(d_6)$

$d_8 = F_i(d_7)$

d_{12} d_8

```
   merge
   ... x ...
   ... y ...
8: y := ...
```

d_{13}, d_{14}, d_{15}, d_{16}

$d_{13} = F_m(d_{12}, d_8)$

$d_{14} = F_n(d_{13})$

$d_{15} = F_o(d_{14})$

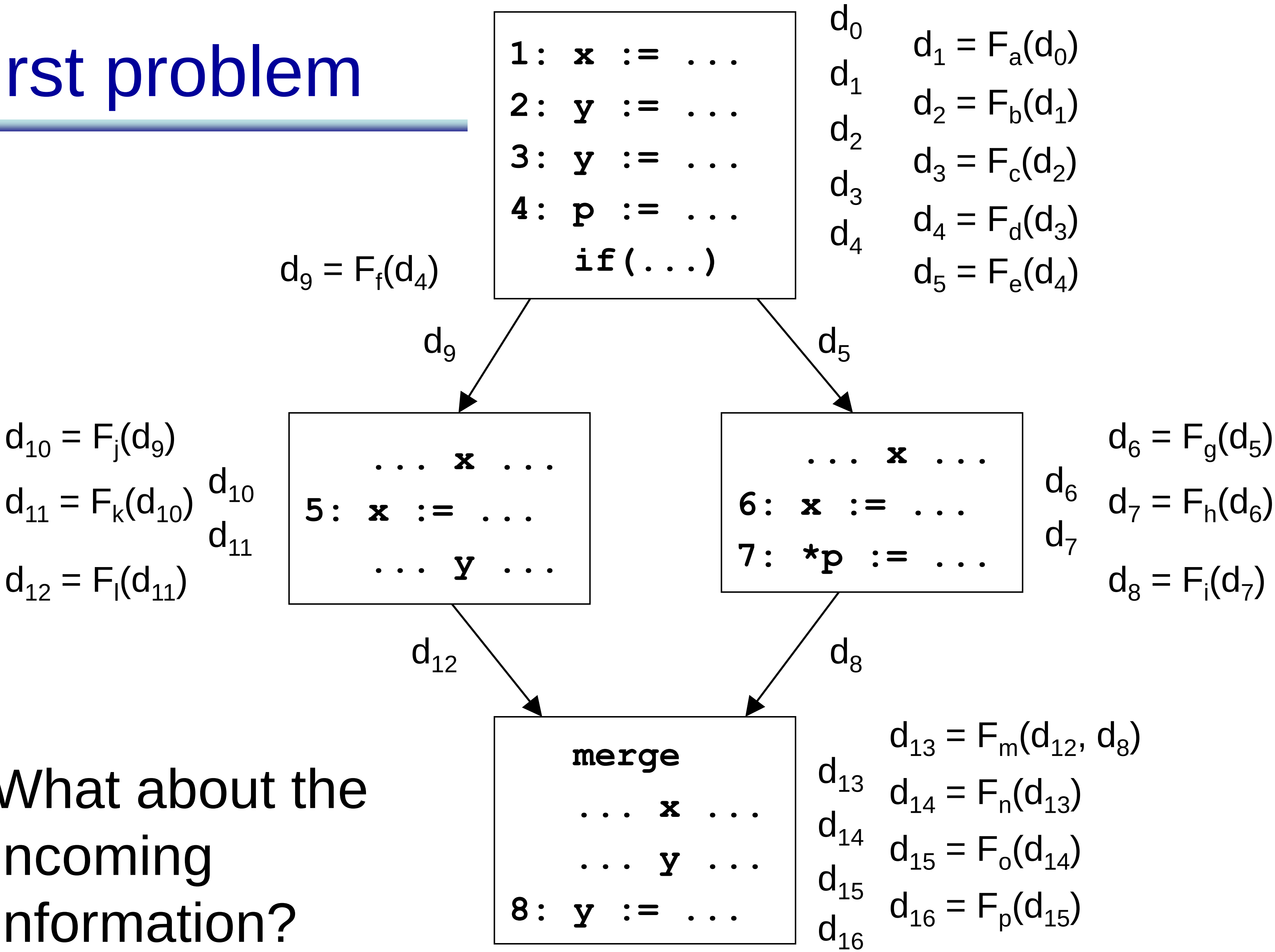
$d_{16} = F_p(d_{15})$

How to find solutions for d_i?

How to find solutions for d_i?

- This is a forward problem
  - given information flowing ***in*** to a node, can determine using the flow function the info flow ***out*** of the node
- To solve, simply propagate information forward through the control flow graph, using the flow functions
- What are the problems with this approach?

First problem

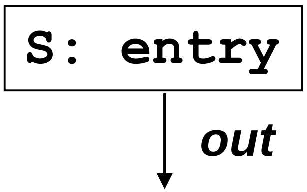


What about the incoming information?

First problem

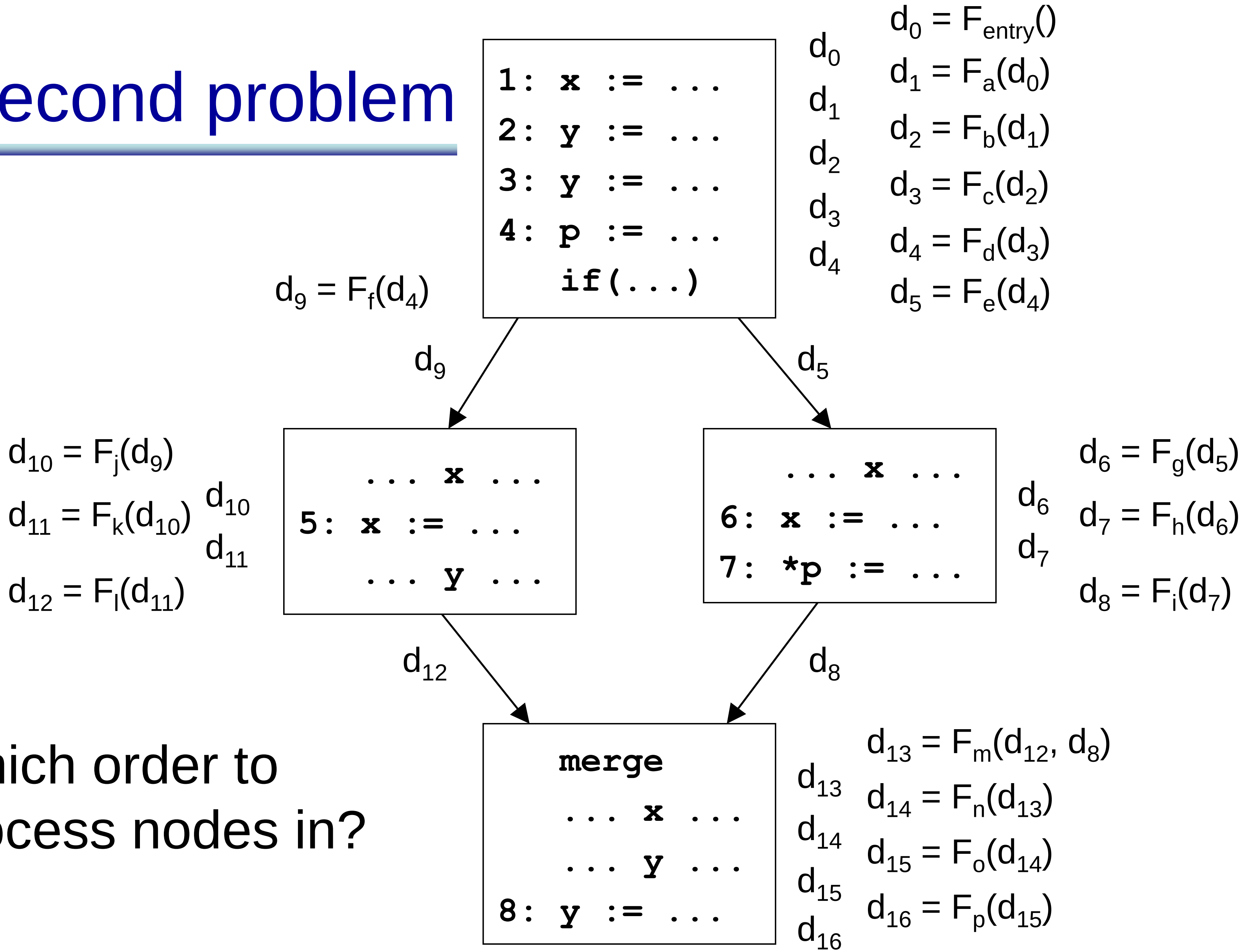
- What about the incoming information?
  - d_0 is not constrained
  - so where do we start?
- Need to constrain d_0
- Two options:
  - explicitly state entry information
  - have an entry node whose flow function sets the information on entry (doesn’t matter if entry node has an incoming edge, its flow function ignores any input)

Entry node



$$out = \{ X \to S \mid X \in \text{Formals} \}$$

Second problem



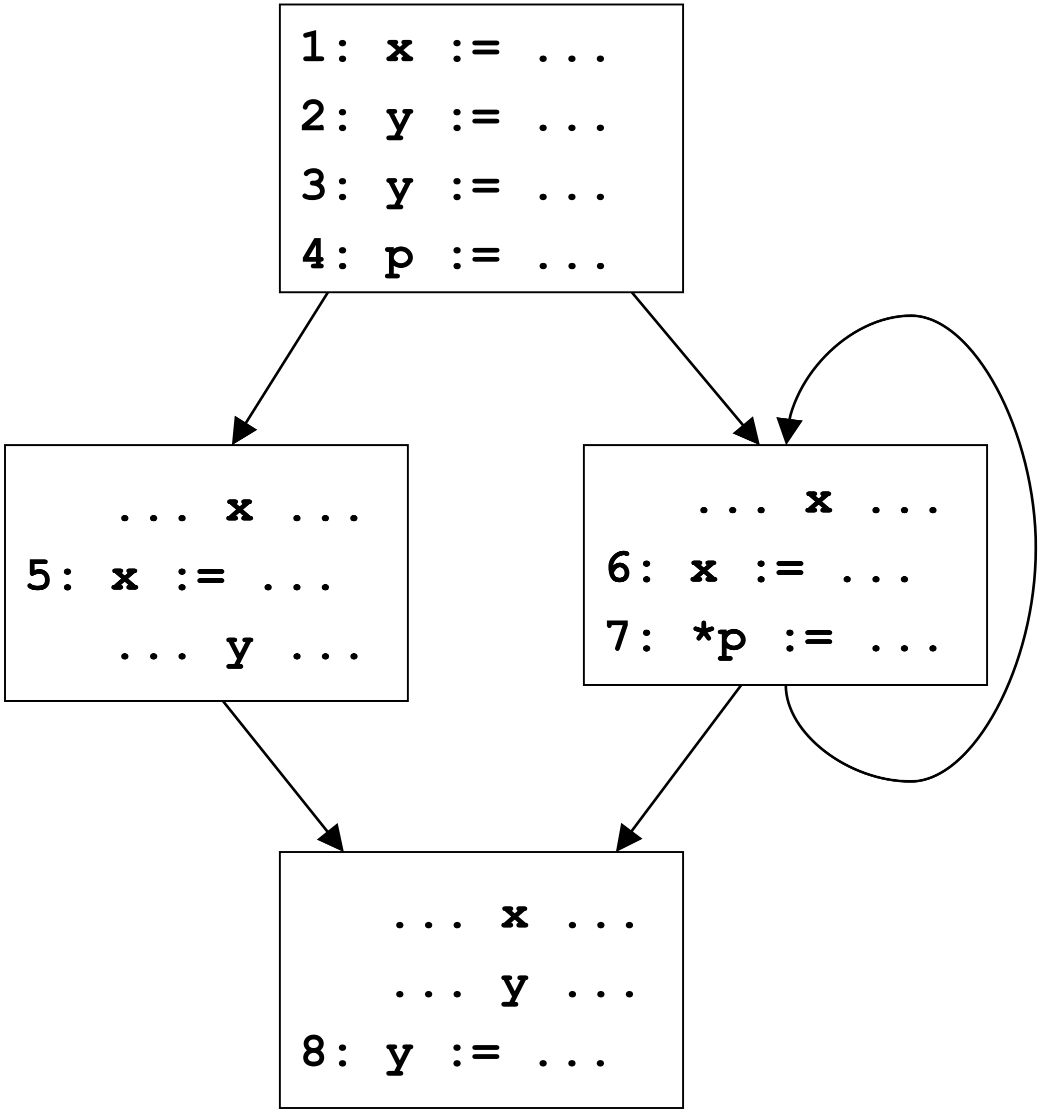
Which order to process nodes in?

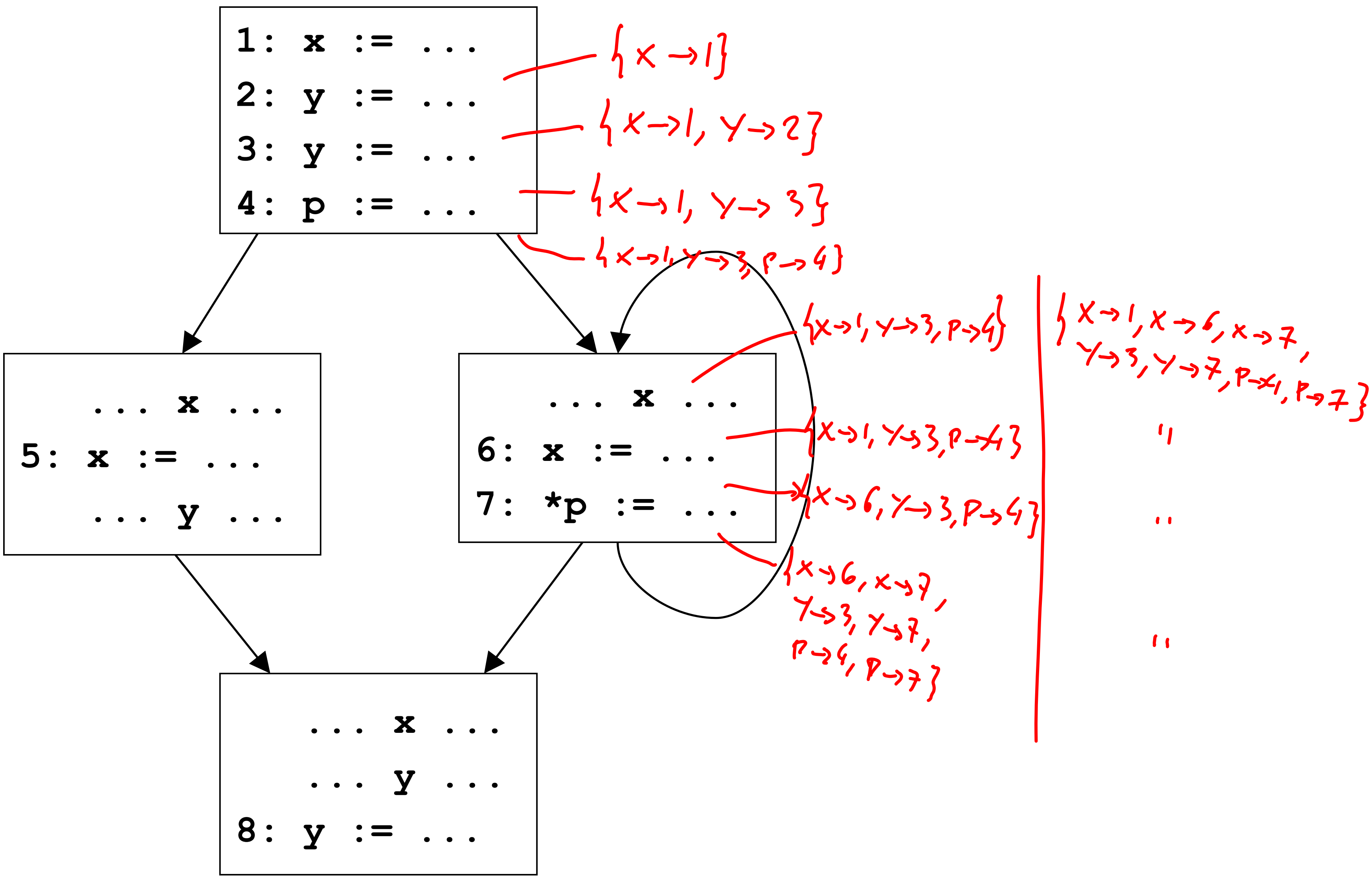
Second problem

- Which order to process nodes in?
- Sort nodes in topological order
  - each node appears in the order after all of its predecessors
- Just run the flow functions for each of the nodes in the topological order
- What’s the problem now?

Second problem, prime

- When there are loops, there is no topological order!
- What to do?
- Let’s try and see what we can do


1: x := ...
2: y := ...
3: y := ...
4: p := ...
... x ...
5: x := ...
... y ...
... x ...
6: x := ...
7: *p := ...
... x ...
... y ...
8: y := ...



1: x := ...
2: y := ...
3: y := ...
4: p := ...
... x ...
5: x := ...
... y ...
... x ...
6: x := ...
7: *p := ...
... x ...
... y ...
8: y := ...
{x→1}
{x→1, y→2}
{x→1, y→3}
{x→1, y→3, p→4}
{x→1, y→3, p→4}
{x→1, y→3, p→4}
{x→6, y→3, p→4}
{x→6, x→7, y→3, y→7, p→4, p→7}
{x→1, x→6, x→7, y→3, y→7, p→4, p→7}
"
"
"

Worklist algorithm

- Initialize all d_i to the empty set
- Store all nodes onto a worklist
- while worklist is not empty:
  - remove node n from worklist
  - apply flow function for node n
  - update the appropriate d_i, and add nodes whose inputs have changed back onto worklist

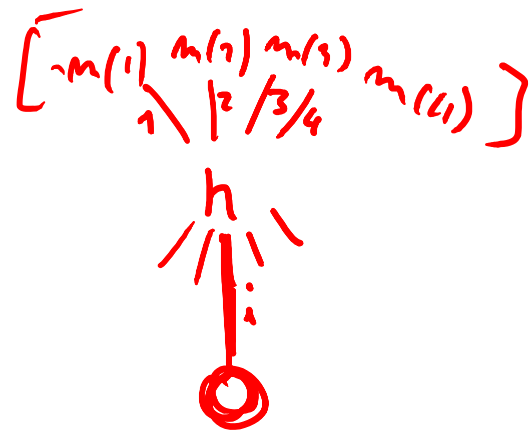
Worklist algorithm

```
let m: map from edge to computed value at edge
let worklist: work list of nodes

for each edge e in CFG do
   m(e) := ∅

for each node n do
   worklist.add(n)

while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length-1 do
      if (m(n.outgoing_edges[i]) ≠ info_out[i])
         m(n.outgoing_edges[i]) := info_out[i];
         worklist.add(n.outgoing_edges[i].dst);
```



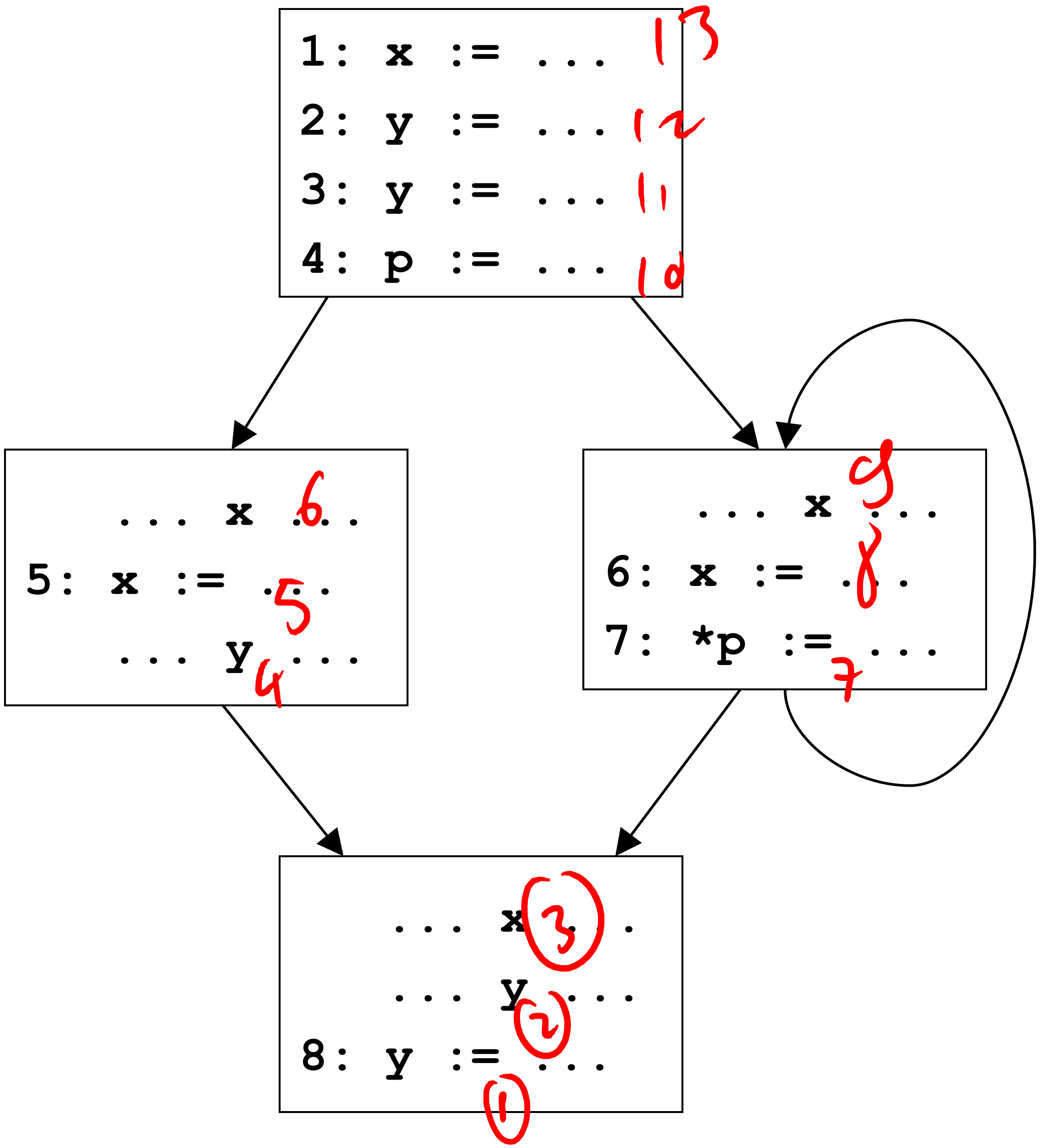
Issues with worklist algorithm

Two issues with worklist algorithm

- Ordering
  - In what order should the original nodes be added to the worklist?
  - What order should nodes be removed from the worklist?
- Does this algorithm terminate?

Order of nodes

- Topological order assuming back-edges have been removed
- Reverse depth-first post-order
- Use an ordered worklist


1: x := ...
2: y := ...
3: y := ...
4: p := ...
13
12
11
10
... x ...
5: x := ...
... y ...
6
5
4
... x ...
6: x := ...
7: *p := ...
9
8
7
... x ...
... y ...
8: y := ...
3
2
1

Termination

- Why is termination important?
- Can we stop the algorithm in the middle and just say we’re done...
- No: we need to run it to completion, otherwise the results are not safe...

Termination

- Assuming we're doing reaching defs, let's try to guarantee that the worklist loop terminates, regardless of what the flow function F does

```
while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length-1 do
      if (m(n.outgoing_edges[i]) ≠ info_out[i])
         m(n.outgoing_edges[i]) := info_out[i];
         worklist.add(n.outgoing_edges[i].dst);
```

Termination

- Assuming we're doing reaching defs, let's try to guarantee that the worklist loop terminates, regardless of what the flow function F does

```
while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length-1 do
      let new_info := m(n.outgoing_edges[i]) ∪
                      info_out[i];
      if (m(n.outgoing_edges[i]) ≠ new_info])
         m(n.outgoing_edges[i]) := new_info;
         worklist.add(n.outgoing_edges[i].dst);
```

Structure of the domain

- We’re using the structure of the domain outside of the flow functions

- In general, it’s useful to have a framework that formalizes this structure

- We will use lattices