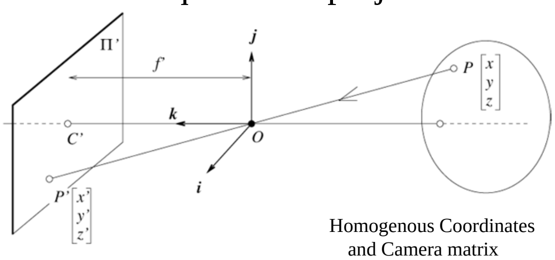
Image Formation
and Cameras
(Part 2)

Introduction to Computer Vision
CSE 152
Lecture 5

Announcements

- Homework 1 is due Apr 24, 11:59 PM
- Wait list
- Reading:
  - Chapter 2 Image formation

The equation of projection



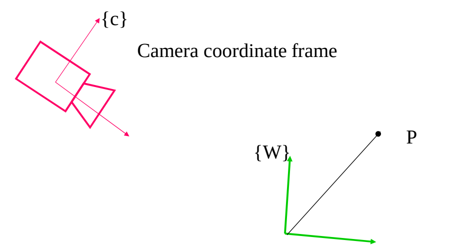
Homogenous Coordinates
and Camera matrix

Cartesian coordinates:

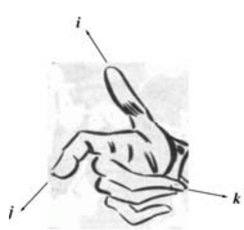
$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

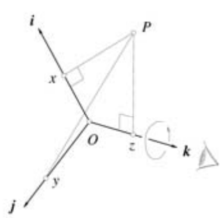
$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

What if camera coordinate system differs from object coordinate system



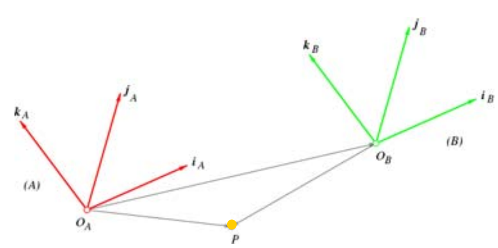
Euclidean Coordinate Systems





$$\begin{cases} x = \overrightarrow{OP}.\mathbf{i} \\ y = \overrightarrow{OP}.\mathbf{j} \\ z = \overrightarrow{OP}.\mathbf{k} \end{cases} \Leftrightarrow \overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Leftrightarrow \mathbf{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Coordinate Changes: Pure Translations

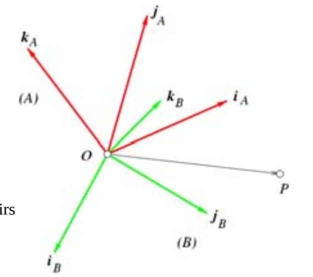


$$\overrightarrow{O_B P} = \overrightarrow{O_A P} + \overrightarrow{O_B O_A}$$

$$\overrightarrow{O_B P} = \overrightarrow{O_A P} + t$$

Translation from coordinate frame A to coordinate frame B

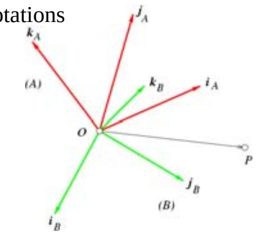
Rotation Matrix



Dot Products between all pairs of coordinate axis of both systems

$$R = \begin{bmatrix} \mathbf{i}_A.\mathbf{i}_B & \mathbf{j}_A.\mathbf{i}_B & \mathbf{k}_A.\mathbf{i}_B \\ \mathbf{i}_A.\mathbf{j}_B & \mathbf{j}_A.\mathbf{j}_B & \mathbf{k}_A.\mathbf{j}_B \\ \mathbf{i}_A.\mathbf{k}_B & \mathbf{j}_A.\mathbf{k}_B & \mathbf{k}_A.\mathbf{k}_B \end{bmatrix} = \begin{bmatrix} \mathbf{i}_B & \mathbf{j}_B & \mathbf{k}_B \end{bmatrix}^T \begin{bmatrix} \mathbf{i}_A & \mathbf{j}_A & \mathbf{k}_A \end{bmatrix}$$

Coordinate Changes: Pure Rotations



Rotation from coordinate frame A to coordinate frame B

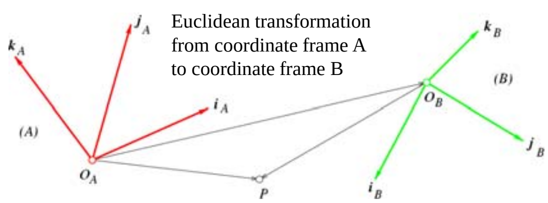
$$\overrightarrow{OP} = \begin{bmatrix} \mathbf{i}_A & \mathbf{j}_A & \mathbf{k}_A \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = \begin{bmatrix} \mathbf{i}_B & \mathbf{j}_B & \mathbf{k}_B \end{bmatrix} \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{i}_A & \mathbf{j}_A & \mathbf{k}_A \end{bmatrix} P_A = \begin{bmatrix} \mathbf{i}_B & \mathbf{j}_B & \mathbf{k}_B \end{bmatrix} P_B$$

$$\Rightarrow P_B = \begin{bmatrix} \mathbf{i}_B & \mathbf{j}_B & \mathbf{k}_B \end{bmatrix}^T \begin{bmatrix} \mathbf{i}_A & \mathbf{j}_A & \mathbf{k}_A \end{bmatrix} P_A = R\,P_A$$

Coordinate Changes: Euclidean Transformations

Euclidean transformation from coordinate frame A to coordinate frame B



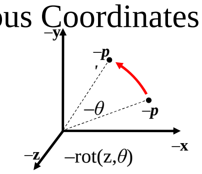
$$P_B = R\,P_A + t$$

3D Rotation Matrices

- $R^{\mathrm{T}} = R^{-1}$
- $R^{\mathrm{T}}R = RR^{\mathrm{T}} = I$
- $\det(R) = 1$
- $R_{i,j} \in [-1, +1]$
- Rows (or columns) of R form a right handed orthonormal coordinate system
- Even though a rotation matrix is 3x3 with nine numbers, it only has three degrees of freedom, it can be parameterized with three numbers. There are many parameterizations

Rotation: Homogenous Coordinates

- About z axis



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation

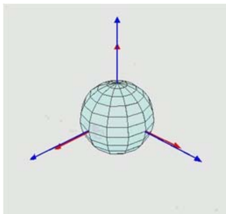
- About x axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- About y axis:

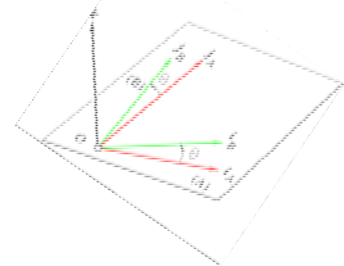
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Composition of Rotations



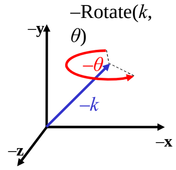
Roll-Pitch-Yaw

$$R = rot(\hat{i},\alpha)rot(\hat{j},\beta)rot(\hat{k},\varphi)$$



Rotation

- About (k_x, k_y, k_z), a unit vector on an arbitrary axis (Rodrigues Formula)



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} k_x k_x(1-c)+c & k_z k_x(1-c)-k_z s & k_x k_z(1-c)+k_y s & 0 \\ k_y k_x(1-c)+k_z s & k_z k_x(1-c)+c & k_y k_z(1-c)-k_x s & 0 \\ k_z k_x(1-c)-k_y s & k_z k_x(1-c)-k_x s & k_z k_z(1-c)+c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

where $c = \cos\theta$ & $s = \sin\theta$

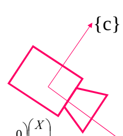
Euclidean Transformations, Homogeneous Coordinates

$$\begin{bmatrix} P_B \\ 1 \end{bmatrix} = \begin{bmatrix} R\,P_A + t \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} P_A \\ 1 \end{bmatrix} = E \begin{bmatrix} P_A \\ 1 \end{bmatrix}$$

Euclidean transformation represented by 4x4 Matrix

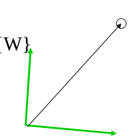
$$E = \begin{bmatrix} R & t \\ \mathbf{0}^T & 1 \end{bmatrix}$$

What if camera coordinate system differs from object coordinate system



Camera coordinate frame

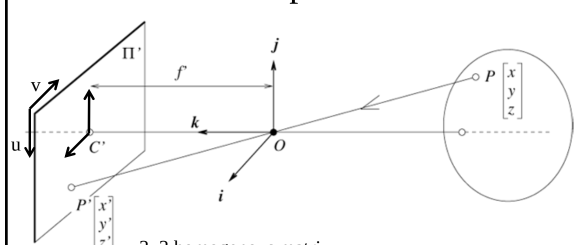
$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



$$E_{w,c} = \begin{bmatrix} R & t \\ \mathbf{0}^T & 1 \end{bmatrix}$$

World coordinate frame

Intrinsic parameters



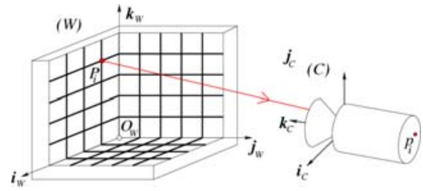
- –3x3 homogenous matrix
- –Focal length:
- –Principal Point: C'
- –Units (e.g. pixels)
- –Orientation and position of image coordinate system
- –Pixel Aspect ratio

Camera parameters

- Extrinsic Parameters: Since camera may not be at the origin, there is a rigid transformation between the world coordinates and the camera coordinates
- Intrinsic parameters: Since scene units (e.g., cm) differ image units (e.g., pixels) and coordinate system may not be centered in image, we capture that with a 3x3 transformation comprised of focal length, principal point, pixel aspect ratio, angle between axes, etc.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \underbrace{\begin{pmatrix} \text{Transformation} \\ \text{represented by} \\ \text{intrinsic parameters} \end{pmatrix}}_{3 \times 3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} \text{Euclidean transformation} \\ \text{represented by} \\ \text{extrinsic parameters} \end{pmatrix}}_{4 \times 4} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

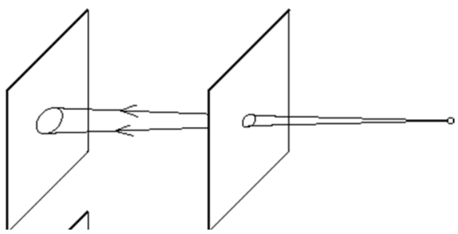
Camera Calibration



Given n points $P_1, \ldots, P_n$ with *known* positions and their images $p_1, \ldots, p_n$ –, estimate intrinsic and extrinsic camera parameters

- See Text book for how to do it.
- Camera Calibration Toolbox for Matlab (Bouguet)
  – http://www.vision.caltech.edu/bouguetj/calib_doc/

Beyond the pinhole Camera
Getting more light – Bigger Aperture

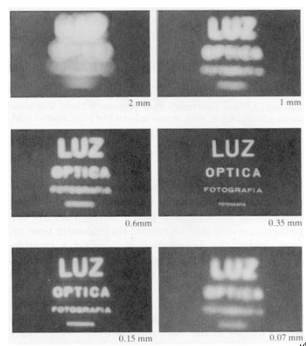


Pinhole Camera Images with Variable Aperture

2 mm **1mm**

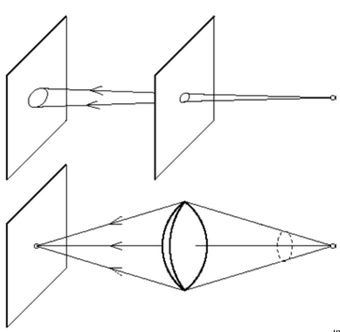
.6 mm **.35 mm**

.15 mm **.07 mm**

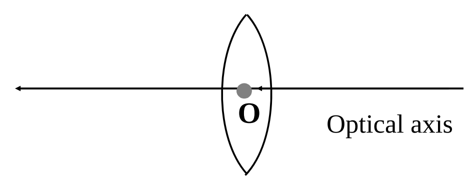


The reason for lenses

We need light, but big pinholes cause blur.

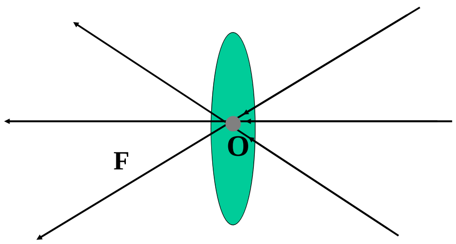


Thin Lens



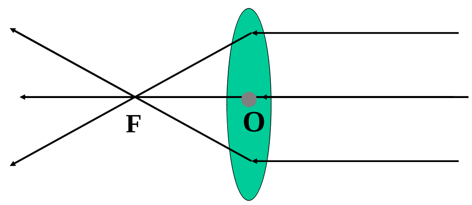
- Rotationally symmetric about optical axis.
- Spherical interfaces.

Thin Lens: Center



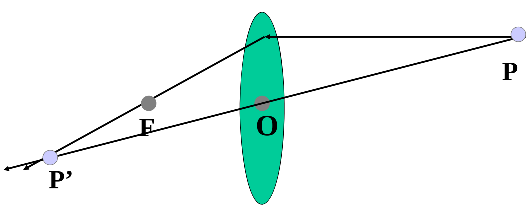
- All rays that enter lens along line pointing at **O** emerge in same direction.

Thin Lens: Focus



Parallel lines pass through the focus, F

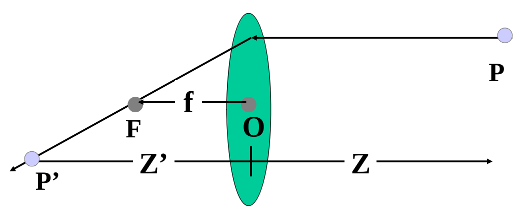
Thin Lens: Image of Point



– All rays passing through lens and starting at **P** converge upon **P’**

– So light gather capability of lens is given the area of the lens and all the rays focus on P’ instead of become blurred like a pinhole

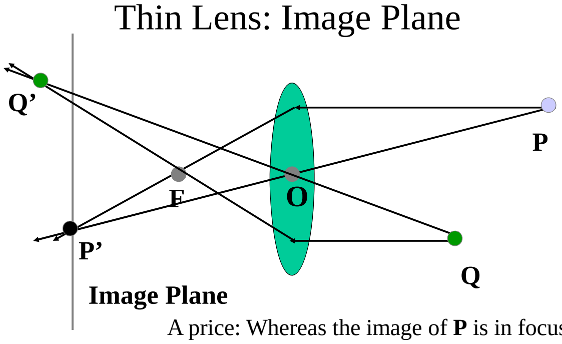
Thin Lens: Image of Point



$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

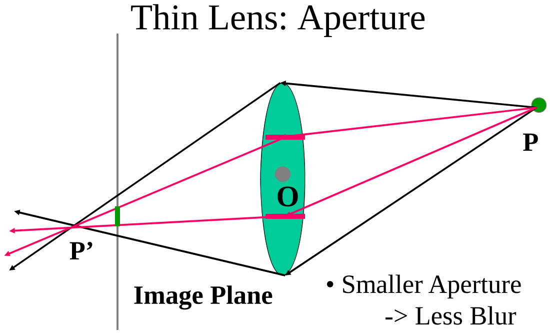
Relation between depth of Point (Z) and the depth where it focuses (Z’)

Thin Lens: Image Plane



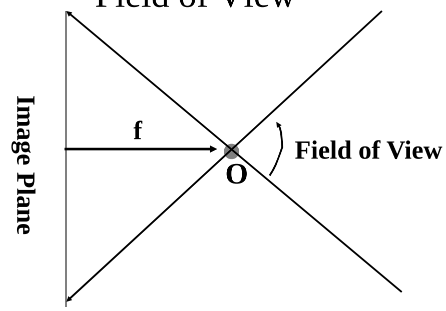
A price: Whereas the image of **P** is in focus, the image of **Q** isn’t.

Thin Lens: Aperture



- Smaller Aperture -> Less Blur
- Pinhole -> No Blur

Field of View



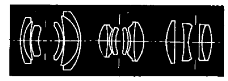
Field of view is a function of **f** and size of image plane.

Deviations from the lens model

Deviations from this ideal are *aberrations*
Two types

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma
2. chromatic

Aberrations are reduced by combining lenses

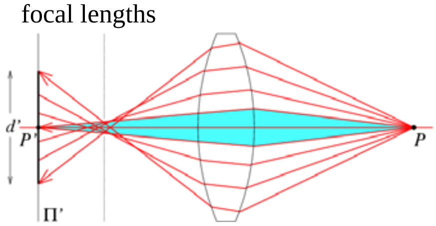


Compound lenses

Spherical aberration

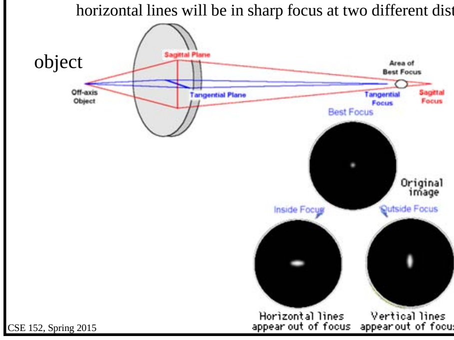
Rays parallel to the axis do not converge

Outer portions of the lens yield smaller focal lengths



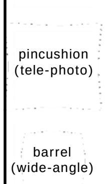
Astigmatism

An optical system with astigmatism is one where rays that propagate in two perpendicular planes have different focus. If an optical system with astigmatism is used to form an image of a cross, the vertical and horizontal lines will be in sharp focus at two different distances.



Distortion

magnification/focal length different for different angles of inclination



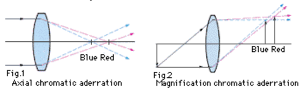
Can be corrected! (if parameters are know)

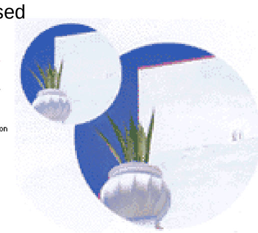
Chromatic aberration

(great for prisms, bad for lenses)

Chromatic aberration

rays of different wavelengths focused in different planes





The image is blurred and appears colored at the fringe.

cannot be removed completely

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Vignetting

– Only part of the light reaches the sensor
– Periphery of the image is dimmer

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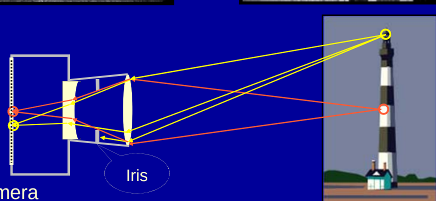
Vignetting: Spatial Non-Uniformity



Litvinov & Schechner, *radiometric nonidealities*