Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 4

Announcements

http://cseweb.ucsd.edu/classes/sp15/cse152-a/

- Piazza
- Homework 1 will be assigned today
- Wait list, additional TA/Tutor, larger room
- Reading:
  - Chapter 2 Image formation

Image Formation: Outline

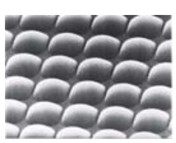
- Factors in producing images
- Projection
- Perspective
- Vanishing points
- Orthographic
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance

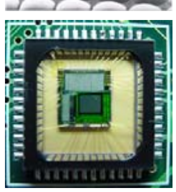
Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness
- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed





Images are two-dimensional patterns of brightness values.

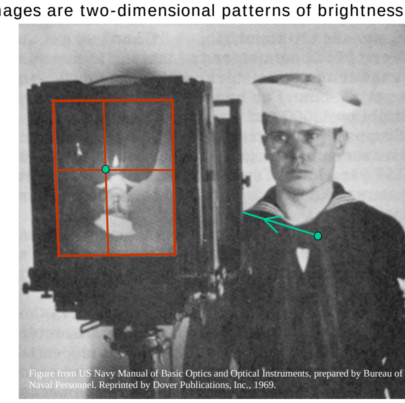


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

They are formed by the projection of 3D objects.

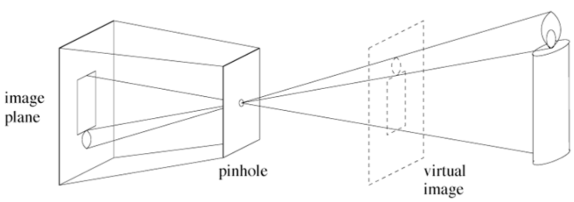
Effect of Lighting: Monet


Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it



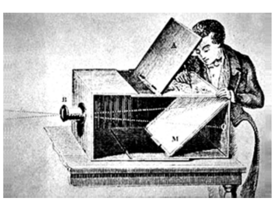
 Forsyth&Ponce

Camera Obscura

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". --- Leonardo *Da Vinci*

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

Camera Obscura



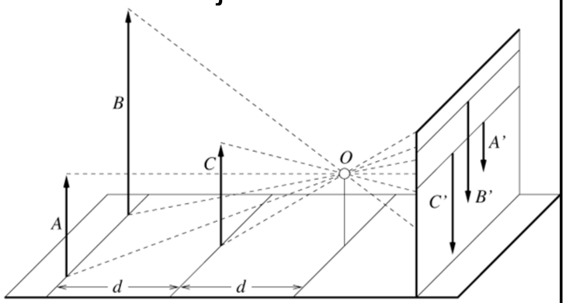
- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

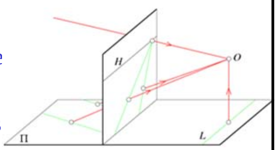
Distant objects are smaller



(Forsyth & Ponce)

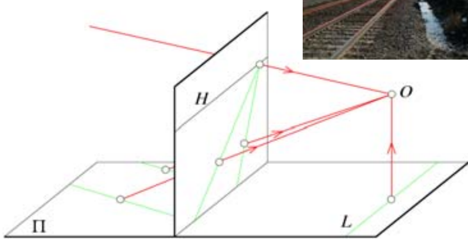
Geometric properties of projection

- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons

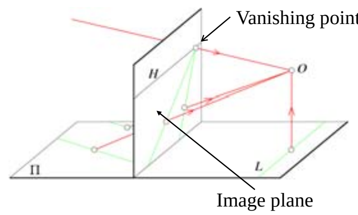


- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Degenerate cases:
  – line through focal point project to point
  – plane through focal point projects to a line

In the perspective image, two parallel lines meet at a point



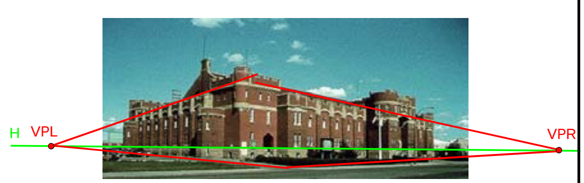
Parallel lines meet in the image



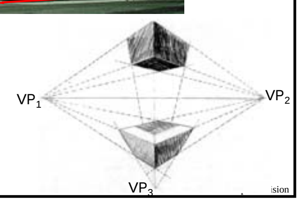
- Formed by line through O
- Parallel to the given line(s)
- A single line can have a vanishing point

Projective geometry provides an elegant means for handling these different situations in a unified way, and **homogenous coordinates** are a way to represent entities (points & lines) in projective spaces.

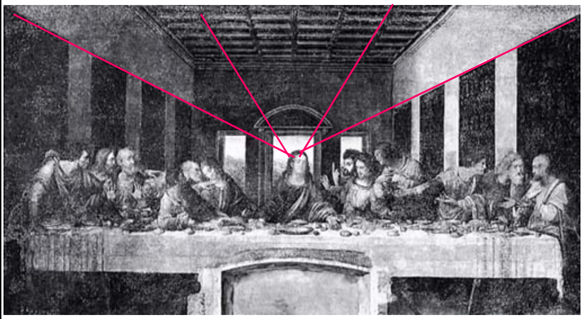
Vanishing points



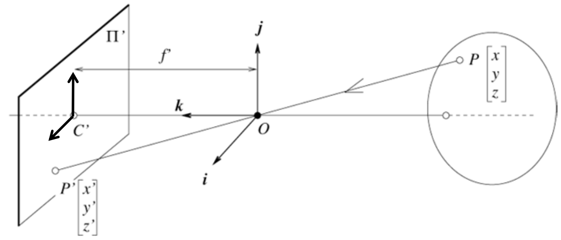
Different directions correspond different vanishing points



Vanishing Points



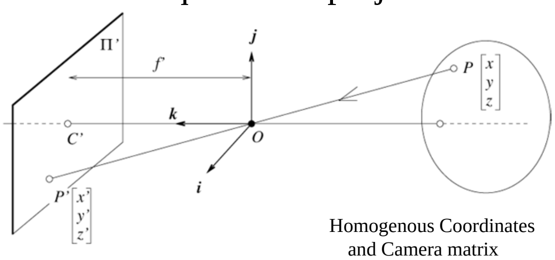
Equation of Perspective Projection



Cartesian coordinates:

- We have, by similar triangles, that (x, y, z) -> (f x/z, f y/z, -f)
- Ignoring the third coordinate, we get $(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$

The equation of projection



Cartesian coordinates:

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogenous Coordinates and Camera matrix

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Simplified Camera Models

Perspective Projection

↓ **Approximation**

Affine Camera Model

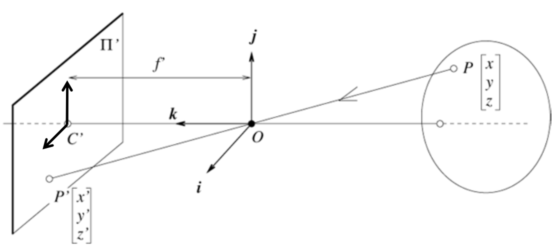
↓

Scaled Orthographic Projection — **Particular case**

↓

Orthographic Projection

Equation of Perspective Projection

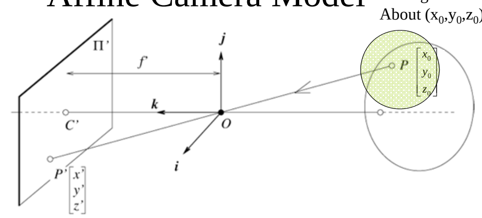


Cartesian coordinates:

- We have, by similar triangles, that (x', y', z') = (f' x/z, f' y/z, f')
- Establishing an image plane coordinate system at C' aligned with i and j, image coordinates of the projection of P are $(x,y,z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$

Affine Camera Model

Appropriate in Neighborhood About (x_0, y_0, z_0)



- Take perspective projection equation, and perform Taylor series expansion about some point P= (x_0, y_0, z_0).
- Drop terms that are higher order than linear.
- Resulting expression is called the affine camera model

• Perspective

$$\begin{bmatrix}u\\v\end{bmatrix}=\frac{f}{z}\begin{bmatrix}x\\y\end{bmatrix}$$

• Perform a Taylor series expansion about (x_0, y_0, z_0)

$$\begin{bmatrix}u\\v\end{bmatrix}=\frac{f}{z_0}\begin{bmatrix}x_0\\y_0\end{bmatrix}-\frac{f}{z_0^2}\begin{bmatrix}x_0\\y_0\end{bmatrix}(z-z_0)+\frac{f}{z_0}\begin{bmatrix}1\\0\end{bmatrix}(x-x_0)$$

$$+\frac{f}{z_0}\begin{bmatrix}0\\1\end{bmatrix}(y-y_0)+\frac{f}{2}\frac{2}{z_0^3}\begin{bmatrix}x_0\\y_0\end{bmatrix}(z-z_0)^2+\cdots$$

• Dropping higher order terms and regrouping.

$$\begin{bmatrix}u\\v\end{bmatrix}\approx\frac{f}{z_0}\begin{bmatrix}x_0\\y_0\end{bmatrix}+\begin{bmatrix}f/z_0 & 0 & -fx_0/z_0^2\\0 & f/z_0 & -fy_0/z_0^2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\mathbf{Ap}+\mathbf{b}$$

Affine camera model in Euclidean Coordinates

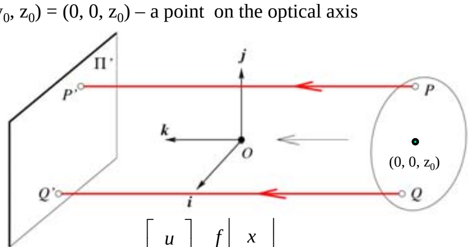
$$\begin{bmatrix}u\\v\end{bmatrix}\approx\frac{f}{z_0}\begin{bmatrix}x_0\\y_0\end{bmatrix}+\begin{bmatrix}f/z_0 & 0 & -fx_0/z_0^2\\0 & f/z_0 & -fy_0/z_0^2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\mathbf{Ap}+\mathbf{b}$$

Rewrite affine camera model
in terms of Homogenous Coordinates

$$\begin{bmatrix}u\\v\\w\end{bmatrix}\approx\begin{bmatrix}f/z_0 & 0 & -fx_0/z_0^2 & fx_0/z_0\\0 & f/z_0 & -fy_0/z_0^2 & fy_0/z_0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Scaled orthographic projection

Starting with Affine Camera Model, take Taylor series about $(x_o, y_0, z_0) = (0, 0, z_0)$ – a point on the optical axis



$$\begin{bmatrix}u\\v\end{bmatrix}=\frac{f}{z_0}\begin{bmatrix}x\\y\end{bmatrix}$$

– That is the z coordinate is dropped, and the image a scaling of the x and y coordinates, where the **scale is $1/z_0$**, the depth of the point of the expansion.

The projection matrix for scaled orthographic projection

$$\begin{pmatrix}U\\V\\W\end{pmatrix}=\begin{pmatrix}f/z_0 & 0 & 0 & 0\\0 & f/z_0 & 0 & 0\\0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}$$

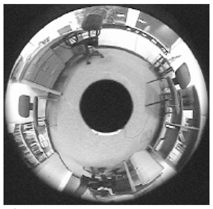
• Parallel lines project to parallel lines
• Ratios of distances are preserved under orthographic projection

For all cameras?

Other camera models

• Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnicam (hemispherical)



Light Probe (spherical)

Some Alternative "Cameras"