Stereo
(Part 2)

Introduction to Computer Vision

CSE 152

Lecture 14

Announcements

- Midterm exam grading will be completed this week
- Homework 3 will be assigned today
- Reading:
  - Section 7.1 Triangulation
  - Section 7.2 Two-frame structure from motion
  - Section 11.1 Epipolar geometry

Two-View Geometry

Essential Matrix E

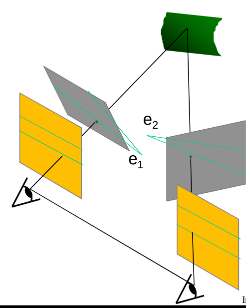
- Rank 2
- Calibrated
- Normalized coordinates
- 5 degrees of freedom
  - Camera rotation
  - Direction of camera translation
- Similarity reconstruction

Fundamental Matrix F

- Rank 2
- Uncalibrated
- Image coordinates
- 7 degrees of freedom
  - Homogeneous matrix to scale
  - det F = 0
- Projective reconstruction

Rectification

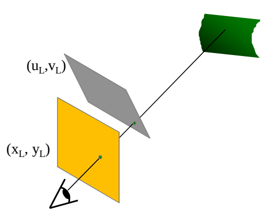
Given a pair of images, transform both images so that epipolar lines are scan lines.



Rectification

Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).

$$\begin{bmatrix} x_L \\ y_L \\ w_L \end{bmatrix} = H_L \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix}$$



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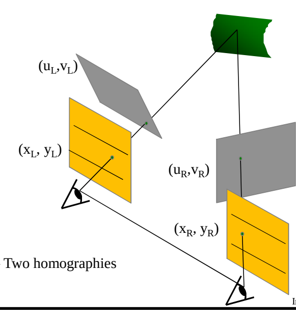


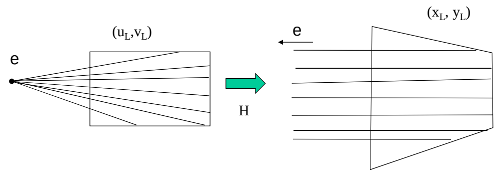
$$\begin{bmatrix} x_R \\ y_R \\ w_R \end{bmatrix} = H_R \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix}$$

Two images – Two homographies

Image pair rectification

Simplify stereo matching by warping the images

Apply projective transformation so that epipolar lines correspond to horizontal scanlines



H should map epipole e to (1,0,0), a point at infinity

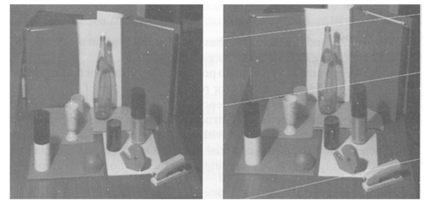
H should minimize image distortion

$$\begin{bmatrix}1\\0\\0\end{bmatrix} = He$$

Note that rectified images usually not rectangular
See Text for complete method

Rectification

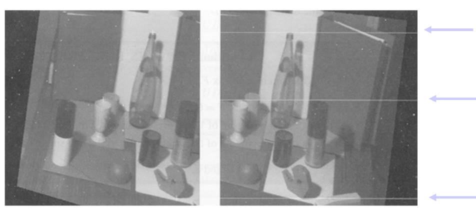
Given a pair of images, transform both images so that epipolar lines are scan lines.



Input Images

Rectification

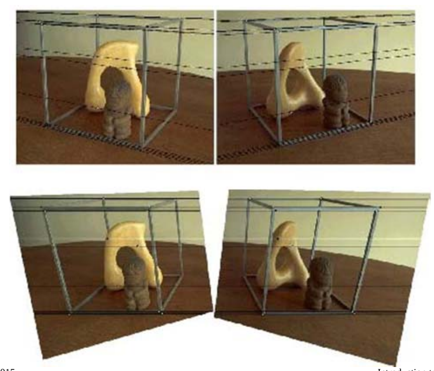
Given a pair of images, transform both images so that epipolar lines are scan lines.



Rectified Images

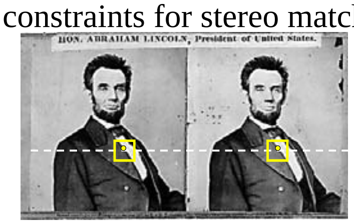
See Section 7.3.7 for specific method

Rectification



Features on same epipolar line

Using epipolar & constant Brightness constraints for stereo matching



For each epipolar line

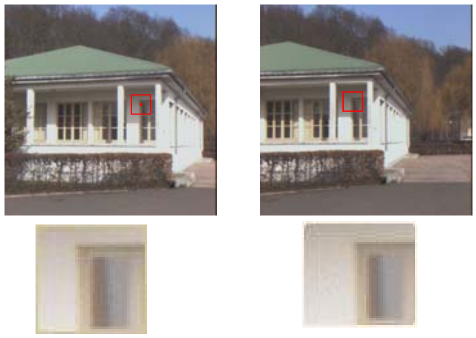
For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost
- This will never work, so:

match ***windows***

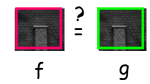
(Seitz)

Finding Correspondences



$W(\mathbf{p}_l)$ $W(\mathbf{p}_r)$

Comparing Windows:



$$SSD = \sum_{[i,j] \in R} (f(i,j) - g(i,j))^2$$

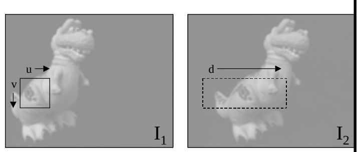
$$C_{fg} = \sum_{[i,j] \in R} f(i,j) g(i,j)$$

Most popular

For each window, match to closest window on epipolar line in other image.

(Camps)

Correspondence Search Algorithm




```
For  i = 1:nrows
  for j=1:ncols
         best(i,j) = -1
         for k = mindisparity:maxdisparity
           c = Match_Metric(I1(i,j),I2(i,j+k),winsize)
           if (c > best(i,j))
                  best(i,j) = c
                  disparities(i,j) = k
           end
         end
  end
end
```

Complexity: O(nrows * ncols * disparities * winx * winy)

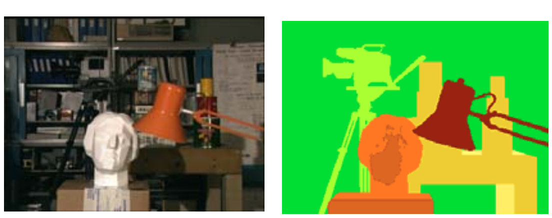
Match Metric Summary

MATCH METRIC	DEFINITION
Normalized Cross-Correlation (NCC)	$\frac{\sum_{u,v}(I_1(u,v)-\bar{I}_1)\cdot(I_2(u+d,v)-\bar{I}_2)}{\sqrt{\sum_{u,v}(I_1(u,v)-\bar{I}_1)^2\cdot\sum_{u,v}(I_2(u+d,v)-\bar{I}_2)^2}}$
Sum of Squared Differences (SSD)	$\sum_{u,v}(I_1(u,v)-I_2(u+d,v))^2$
Normalized SSD	$\sum_{u,v}\left(\frac{(I_1(u,v)-\bar{I}_1)}{\sqrt{\sum_{u,v}(I_1(u,v)-\bar{I}_1)^2}}-\frac{(I_2(u+d,v)-\bar{I}_2)}{\sqrt{\sum_{u,v}(I_2(u+d,v)-\bar{I}_2)^2}}\right)^2$
Sum of Absolute Differences (SAD)	$\sum_{u,v}\lvert I_1(u,v)-I_2(u+d,v)\rvert$
Zero Mean SAD	$\sum_{u,v}\lvert(I_1(u,v)-\bar{I}_1)-(I_2(u+d,v)-\bar{I}_2)\rvert$
Rank	$I'_k(u,v)=\sum_{m,n}I_k(m,n)<I_k(u,v)$ $\sum_{u,v}(I'_1(u,v)-I'_2(u+d,v))$
Census	$I'_k(u,v)=BITSTRING_{m,n}(I_k(m,n)<I_k(u,v))$ $\sum_{u,v}HAMMING(I'_1(u,v),I'_2(u+d,v))$

These two are actually the same

Stereo results

– Data from University of Tsukuba

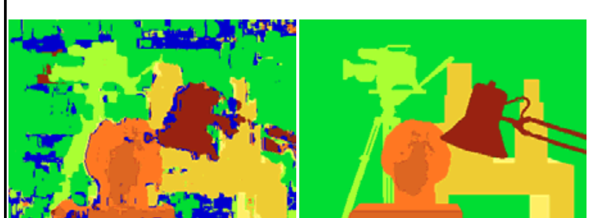


Scene Ground truth

Color coded. For each pixel, green is further and red is closer

(Seitz)

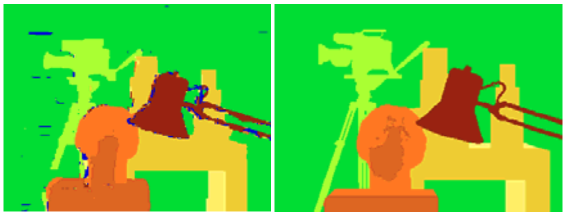
Results with window correlation



Window-based matching (best window size) Ground truth

(Seitz)

Results with better method



Using global optimization — Ground truth

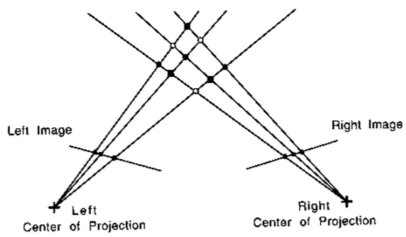
Boykov et al., Fast Approximate Energy Minimization via Graph Cuts, International Conference on Computer Vision, September 1999.

(Seitz)

Some Issues

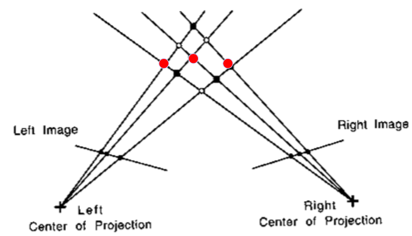
- Epipolar ordering
- Ambiguity
- Window size
- Window shape
- Lighting
- Half occluded regions

A challenge: Multiple Interpretations



Each feature on left epipolar line match one and only one feature on right epipolar line.

Multiple Interpretations



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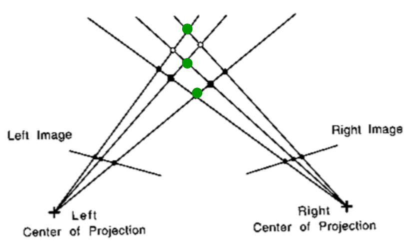
Multiple Interpretations



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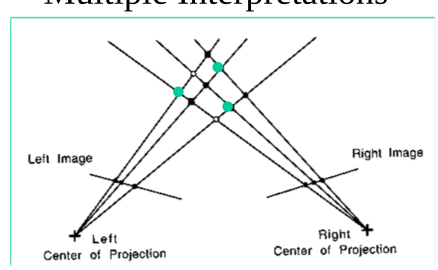
Multiple Interpretations



Each feature on left epipolar line match one and only one feature on right epipolar line.

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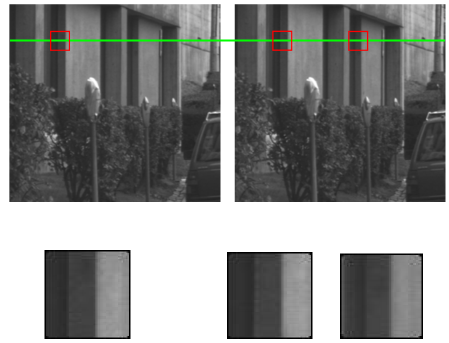
Ambiguity



Some Issues

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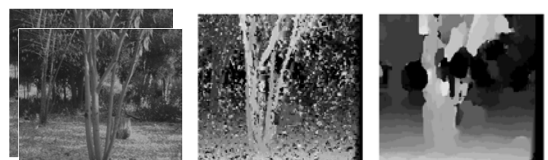
Window size



W = 3 W = 20

- Effect of window size

Better results with *adaptive window*

- T. Kanade and M. Okutomi, *A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment*,, Proc. International Conference on Robotics and Automation, 1991.
- D. Scharstein and R. Szeliski. Stereo matching with nonlinear diffusion. International Journal of Computer Vision, 28(2):155-174, July 1998

(Seitz)

Some Issues

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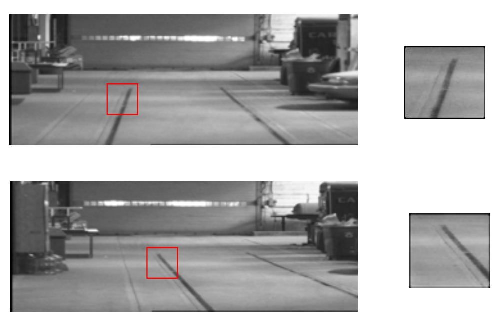
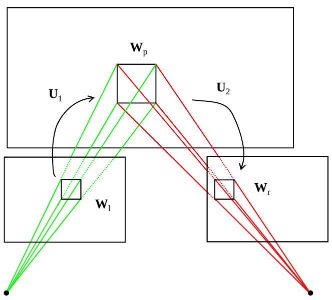
Window Shape and Forshortening



Window Shape: Fronto-parallel Configuration



Some Issues

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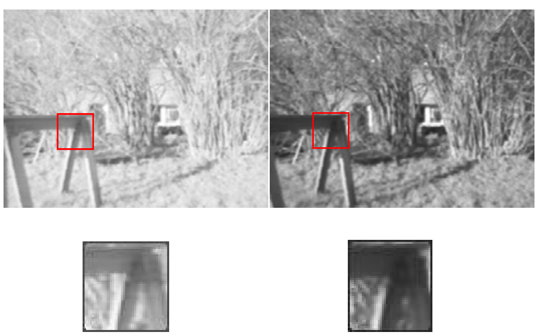
Lighting Conditions (Photometric Variations)



$W(\mathbf{P}_l)$ $W(\mathbf{P}_r)$

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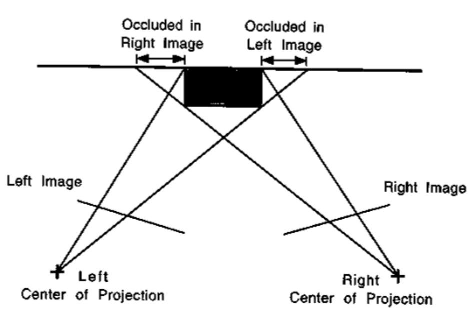
Half occluded regions

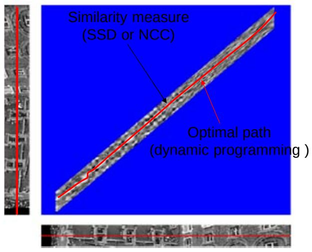


Summary of Stereo Constraints

CONSTRAINT	BRIEF DESCRIPTION
1-D Epipolar Search	Arbitrary images of the same scene may be rectified based on epipolar geometry such that stereo matches lie along one-dimensional scanlines. This reduces the computational complexity and also reduces the likelihood of false matches.
Monotonic Ordering	Points along an epipolar scanline appear in the same order in both stereo images, assuming that all objects in the scene are approximately the same distance from the cameras.
Image Brightness Constancy	Assuming Lambertian surfaces, the brightness of corresponding points in stereo images are the same.
Match Uniqueness	For every point in one stereo image, there is at most one corresponding point in the other image.
Disparity Continuity	Disparities vary smoothly (i.e. disparity gradient is small) over most of the image. This assumption is violated at object boundaries.
Disparity Limit	The search space may be reduced significantly by limiting the disparity range, reducing both computational complexity and the likelihood of false matches.
Fronto-Parallel Surfaces	The implicit assumption made by area-based matching is that objects have fronto-parallel surfaces (i.e. depth is constant within the region of local support). This assumption is violated by sloping and creased surfaces.
Feature Similarity	Corresponding features must be similar (e.g. edges must have roughly the same length and orientation).
Structural Grouping	Corresponding feature groupings and their connectivity must be consistent.

(From G. Hager)

Stereo matching



Constraints
- epipolar
- ordering
- uniqueness
- disparity limit
- disparity gradient limit

Trade-off
- Matching cost (data)
- Discontinuities (prior)

(From Pollefeys)

(Cox et al. CVGIP'96; Koch'96; Falkenhagen´97;
Van Meerbergen,Vergauwen,Pollefeys,VanGool IJCV'02)