Hough transform and line fitting

Introduction to Computer Vision
CSE 152
Lecture 11

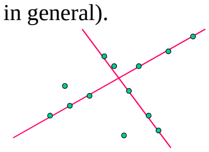
Announcements

- Midterm Exam on Thursday
  - We'll review today
- Homework 2 is due May 12, 11:59 PM

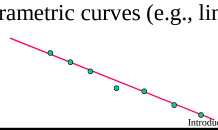
- Reading:
  - Section 4.2 Edges
  - Section 4.3 Lines

What to do with edges?

- Segment linked edge chains into curve features (e.g., line segments).
- Group unlinked or unrelated edges into lines (or curves in general).

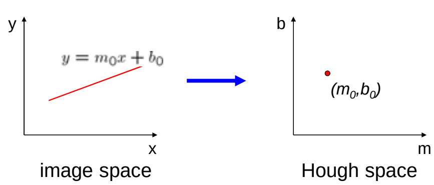


- Accurately fitting parametric curves (e.g., lines) to grouped edge points.



Hough Transform
[Patented 1962]

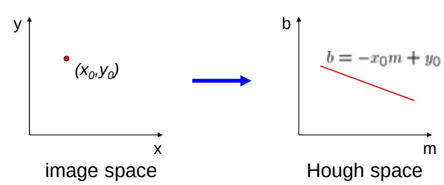
Finding lines in an image



Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space

Finding lines in an image



Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- What does a point (x_0, y_0) in the image space map to?

The equation of any line passing through (x_0, y_0) has form
$y_0 = mx_0 + b$
This is a line in Hough space: $b = -x_0m + y_0$

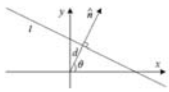
Hough Transform Algorithm

- Typically use a different parameterization

  $d = x\cos\theta + y\sin\theta$

  – d is the perpendicular distance from the line to the origin

  – θ is the angle this perpendicular makes with the x axis



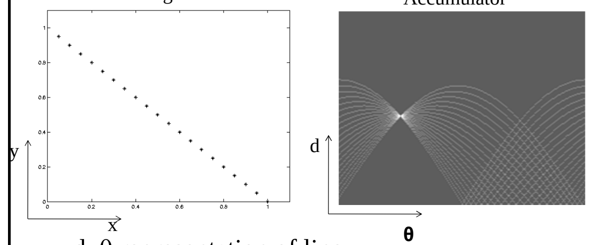
- Basic Hough transform algorithm
  1. Initialize H[d, θ]=0 ; H is called accumulator array
  2. for each edge point I[x,y] in the image

     for θ = 0 to 180

     $d = x\cos\theta + y\sin\theta$

     H[d, θ] += 1
  3. Find the value(s) of (d, θ) where H[d, θ] is the global maximum
  4. The detected line in the image is given by $d = x\cos\theta + y\sin\theta$
- What’s the running time (measured in # votes)?

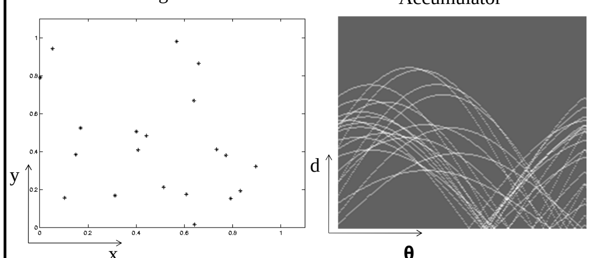
Hough Transform: 20 colinear points



- d, θ representation of line
- Drawn as: $d = |x\cos\theta + y\sin\theta|$
- Maximum accumulator value is 20

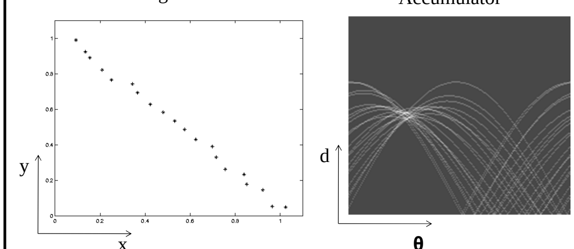
Note: accumulator array range: theta: 0-360, d: positive

Hough Transform: Random points



- d, θ representation of line
- Drawn as: $d = |x\cos\theta + y\sin\theta|$
- Maximum accumulator value is 4

Hough Transform: “Noisy line”



- d, θ representation of line
- Drawn as: $d = |x\cos\theta + y\sin\theta|$
- Maximum accumulator value is 6

Extension: Oriented Edges

procedure *Hough*($\{(x, y, \theta)\}$):

1. Clear the accumulator array.
2. For each detected edgel at location (x, y) and orientation $\theta = \tan^{-1} n_y/n_x$, compute the value of

   $$d = x n_x + y n_y$$

   and increment the accumulator corresponding to (θ, d).
3. Find the peaks in the accumulator corresponding to lines.
4. Optionally re-fit the lines to the constituent edgels.

Algorithm 4.2 Outline of a Hough transform algorithm based on oriented edge segments.

Hough Transform for Curves

Generalized Hough Transform

- The Hough transform can be generalized to detect any curve that can be expressed in parametric form:

  $y = f(x, a_1, a_2, \dots a_p)$

  or

  $g(x, y, a_1, a_2, \dots a_p) = 0$

  – $a_1, a_2, \dots a_p$ are the parameters

  – The parameter space is p-dimensional

  – The accumulating array is *large*

Example: Finding circles

Equation for circle is

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

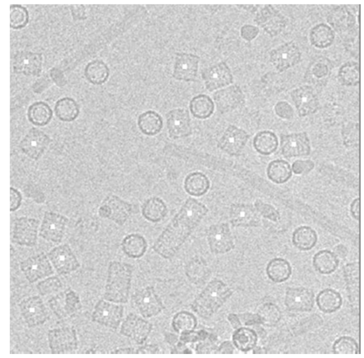
where the parameters are the circle's center (x_c, y_c) and radius r.

Three dimensional generalized Hough space.

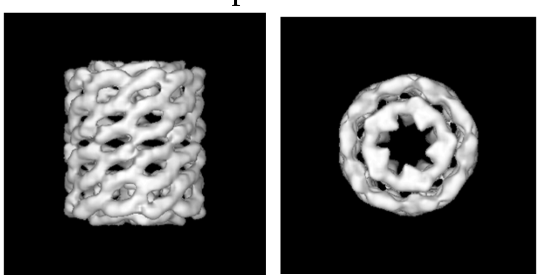
Given an edge point (x,y),

1. Loop over all values of (x_c, y_c),
2. Compute r
3. Increment $H(x_c, y_c, r)$

Transmission Electron Microscopy (TEM) Image of Keyhole Limpet Hemocyanin (KLH) with detected particles



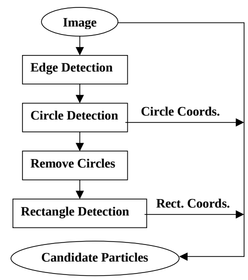
3D Maps of KLH



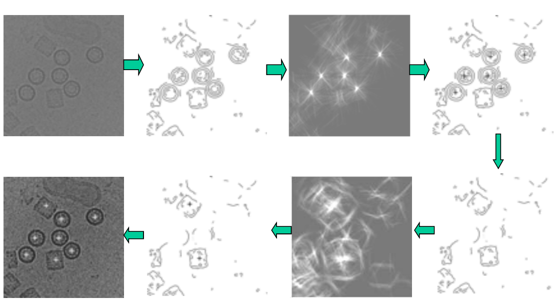
Three-dimensional maps of KLH at a resolution of 23.5 Å reconstructed using particles extracted either manually or automatically as described in the text. (a), (b) The side- and top- view of a 3D map reconstructed from a set of 1042 manually selected particle images.

Processing in Stage 1 for KLH

- Canny edge detection.
- A sequence of ordered Hough transforms (HTs) is applied in order from the computationally simplest one to the most complex one.
- Edges covered by the detected shapes are removed immediately from edge images following the application of the last HT.

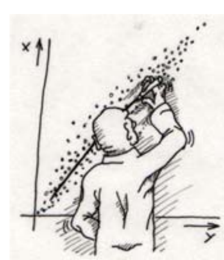


Picking KLH Particles in Stage 1

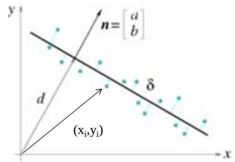


Zhu et al., IEEE Transactions on Medical Imaging, 2003

Line Fitting



Line Fitting



Given n points (x_i, y_i), estimate parameters of line

$$ax_i + by_i - d = 0$$

subject to the constraint that

$$a^2 + b^2 = 1$$

Note: $ax_i + by_i - d$ *is distance from* (x_i, y_i) to line.

Problem: minimize

$$E(a,b,d) = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

Cost Function:
Sum of squared distances between each point and the line

with respect to (a,b,d).

1. Minimize E with respect to d:

$$\frac{\partial E}{\partial d} = 0 \Rightarrow d = \frac{1}{n}\sum_{i=1}^{n} ax_i + by_i = a\bar{x} + b\bar{y}$$

Where $(\bar{x}, \bar{y})$ is the mean of the data points

Line Fitting

2. Substitute d back into E

$$E = \sum_{i=1}^{n} [a(x_i - \bar{x}) + b(y_i - \bar{y})]^2 = |\mathcal{U}\mathbf{n}|^2$$

where $\mathcal{U} = \begin{pmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \dots & \dots \\ x_n - \bar{x} & y_n - \bar{y} \end{pmatrix}$

where $\mathbf{n} = (a\ b)^T$.

3. Minimize $E = |Un|^2 = \mathbf{n}^T U^T U \mathbf{n} = \mathbf{n}^T S \mathbf{n}$ with respect to a, b subject to the constraint $\mathbf{n}^T\mathbf{n} = 1$. Note that S is given by

$$S = \mathcal{U}^T\mathcal{U} = \begin{pmatrix} \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 & \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} \\ \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} & \sum_{i=1}^{n} y_i^2 - n\bar{y}^2 \end{pmatrix}$$

And it's a real, symmetric, positive definite

Line Fitting

4. This is a constrained optimization problem in $\mathbf{n}$. Solve with Lagrange multiplier

$$L(\mathbf{n}) = \mathbf{n}^T S \mathbf{n} - \lambda(\mathbf{n}^T\mathbf{n} - 1)$$

Take partial derivative (gradient) w.r.t. $\mathbf{n}$ and set to 0.

$$\nabla L = 2S\mathbf{n} - 2\lambda\mathbf{n} = 0$$

or

$$S\mathbf{n} = \lambda\mathbf{n}$$

$\mathbf{n} = (a,b)$ is an Eigenvector of the symmetric matrix S (the one corresponding to the smallest Eigenvalue).

5. d is computed from Step 1.

Midterm: Thursday, May 7

- In class, full period
- Coverage
  - Lecture material
    - Slides and anything written on the board
  - Readings
  - Homework 1
  - Homework 2 (lecture material and readings)
- Sheet of notes
  - One sided sheet, hand written
- No calculators

Incomplete list of topics covered

- Human visual system
  - Physiology – from eye to brain
  - Phenomenological
  - Function
- Camera models
- Factors in producing images
- Projection models
  - Perspective
  - Orthographic
- Homogenous Coordinates
- Vanishing points
- Lenses, Distortion
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance
  - BRDF
  - Lambertian
  - Specular
- Color
  - Light Spectrum
  - Reflectance, source
  - Sensor response
  - Color spaces
    - RGB
    - YUV
  - Chromaticity

Incomplete list of topics covered

- Binary Vision
  - Thresholding
  - Neighborhoods
  - Connected component exploration
  - Features, moments
- Noise
  - Additive, Gaussian noise
- Filtering, linear, convolution with Kernel
  - Averaging/smoothing
  - Sharpening
  - Derivatives
  - Gaussian filter
  - Seperability
- Edges & Edge detection
- Edge sources
- Canny
  - Gaussian derivatives
  - Magnitude, orientation
  - Non-maximal supprression
  - Linking/thresholding
- Hough Transform and line fitting