CSE 130: Programming Languages

Environments & Closures

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News

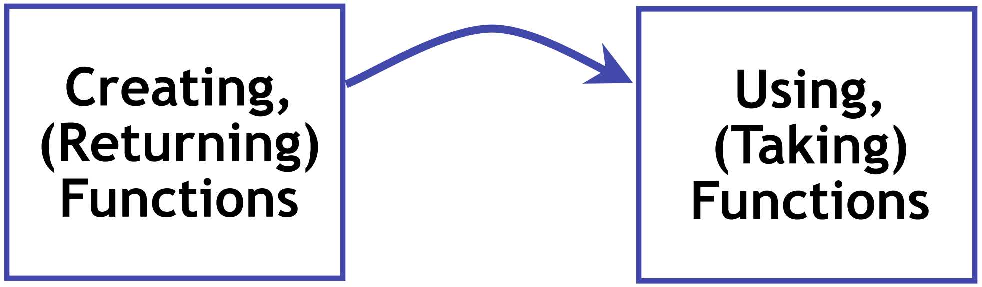
- PA 3 due **THIS** Friday (**5/1**)

- Midterm **NEXT Friday (5/8)**

Recap: Functions as "first-class" values

- Arguments, return values, bindings …
- What are the benefits ?

Parameterized, similar functions (e.g. Testers)



Creating, (Returning) Functions

Using, (Taking) Functions

Iterator, Accumul, Reuse computation pattern w/o exposing local info

Functions are “first-class” values

- Arguments, return values, bindings …
- What are the benefits ?

Parameterized,
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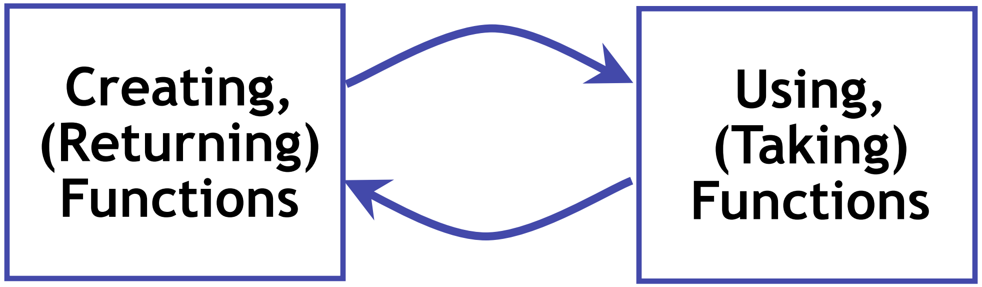


Creating,
(Returning)
Functions

Using,
(Taking)
Functions

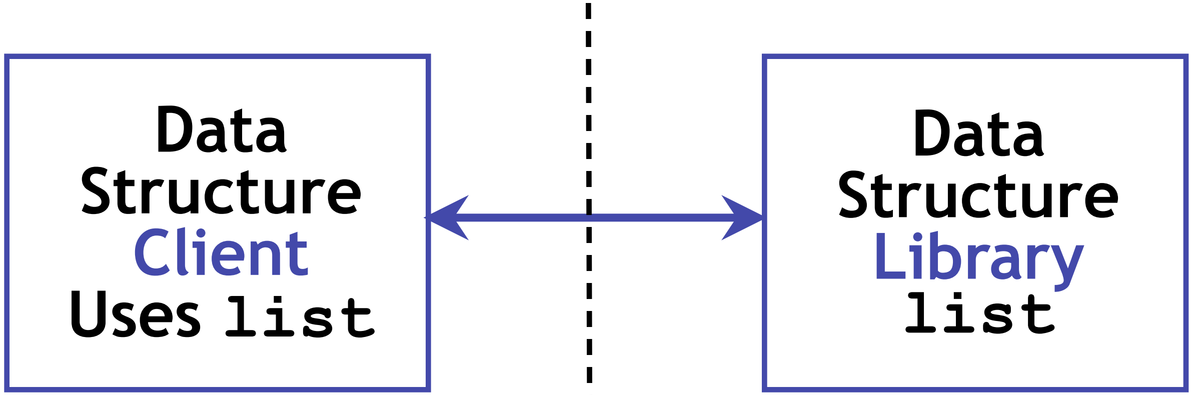
Iterator, Accumul,
Reuse computation
pattern w/o
exposing local info

Compose Functions:
Flexible way to build
Complex functions
from primitives.

Funcs taking/returning funcs

Higher-order funcs enable modular code

- Each part only needs local information



Data Structure Client Uses `list`

Data Structure Library `list`

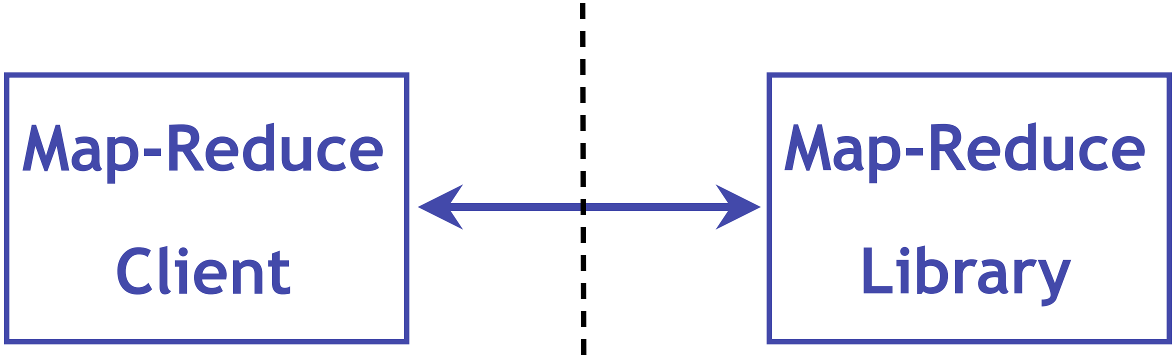
Uses meta-functions:
`map,fold,filter`
With locally-dependent funs
`(lt h), square` etc.
Without requiring Implement.
details of data structure

Provides meta-functions:
`map,fold,filter`
to traverse, accumulate over
lists, trees etc.
Meta-functions don't need client
info (tester ? accumulator ?)

“Map-Reduce” et al.

Higher-order funcs enable modular code

- Each part only needs local information



Map-Reduce

Client

Map-Reduce

Library

Web Analytics “Queries”
Clustering, Page Rank, etc
as map/reduce + **ops**

Provides: **map, reduce**
to traverse, accumulate
over WWW (“Big Data”)
Distributed across “cloud”

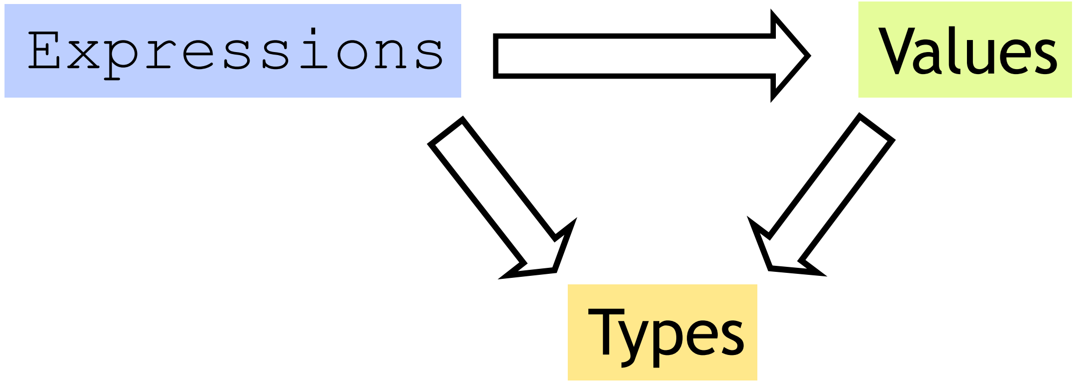
Higher Order Functions

Are Awesome…

Higher Order Functions

..but how do they work

Next: Environments & Functions



Lets start with the humble variable…

Variables and Bindings

Q: How to use variables in ML ?

Q: How to “assign” to a variable ?

```
# let x = 2+2;;
val x : int = 4
```

```
let x = e;;
```

“**Bind** value of **expr** `e` to **variable** `x`”

Variables and Bindings

```
# let x = 2+2;;
val x : int = 4
# let y = x * x * x;;
val y : int = 64
# let z = [x;y;x+y];;
val z : int list = [4;64;68]
```

Later expressions can **use** `x`

- **Most recent** “bound” value used for evaluation

Sounds like C/Java ?

NO!

Environments ("Phone Book")

How ML deals with variables

- Variables = "names"
- Values = "phone number"

'w, Queensbury	01274 881373
Road, Bradford	01274 603920
d, Brighouse	01484 722933
ster Rd, Linthwaite	01484 844586
o, BD6	01274 679404
Slaithwaite	01484 843163
d, Wyke	01274 675753
Slaithwaite	01484 843681
, Queensbury	01274 818683
larsden	01484 844450
ott, Plains, Marsden	01484 844996
layton	01274 816057
ie, Linthwaite	01484 846885
Gro, Cross Roads	01535 643681
l, Todmorden	01706 818413
Av, Bradford	01274 672644
Dv, Queensbury	01274 818887
v, Pellon	01422 259543
Rd, Sowerby Bdge	01422 839907
v, Beechwood	01422 831577
t, Clayton	01274 882408
<, Brighouse	01484 714532

P	10 Prospect Vw,
PJ	22 Shelf Moor Rc
R	5 Arnold Royd, B
R	1041 Mancheste
R	9 St Pauls Gro, B
R	10 Varley Rd, Sla
R	156 Wilson Rd, V
Robert	1 Wood St, Sla
RA	2 Cheriton Dv, Q
RA	5 Dirker Dv, Mar
RB	Dirker Bank Cott,
RC	16 Holts La, Clay
RD	46 Stones Lane,
RW	37 Laburnum Gr
S	160 Bacup Rd, T
S	35 Markfield Av,
SP	9 Brambling Dv,
T	22b Albert Vw, Pe
T	13 Industrial Rd,
TE	39 Whitley Av, Be
V	17 Gregory Ct, Cl
W	43 Bolehill Pk, Bri

...	...
x	4 : int
y	64 : int
z	[4;64;68] : int list
x	8 : int

Environments and Evaluation

ML begins in a “top-level” environment

- Some names bound (e.g. +,-, print_string...)

```
let x = e;;
```

ML program = Sequence of variable bindings

Program evaluated by evaluating bindings in order

1. Evaluate expr **e** in current env to get value *v* : *t*
2. Extend env to bind **x** to *v* : *t*

(Repeat with next binding)

Environments

"Phone book"

- Variables = "names"
- Values = "phone number"

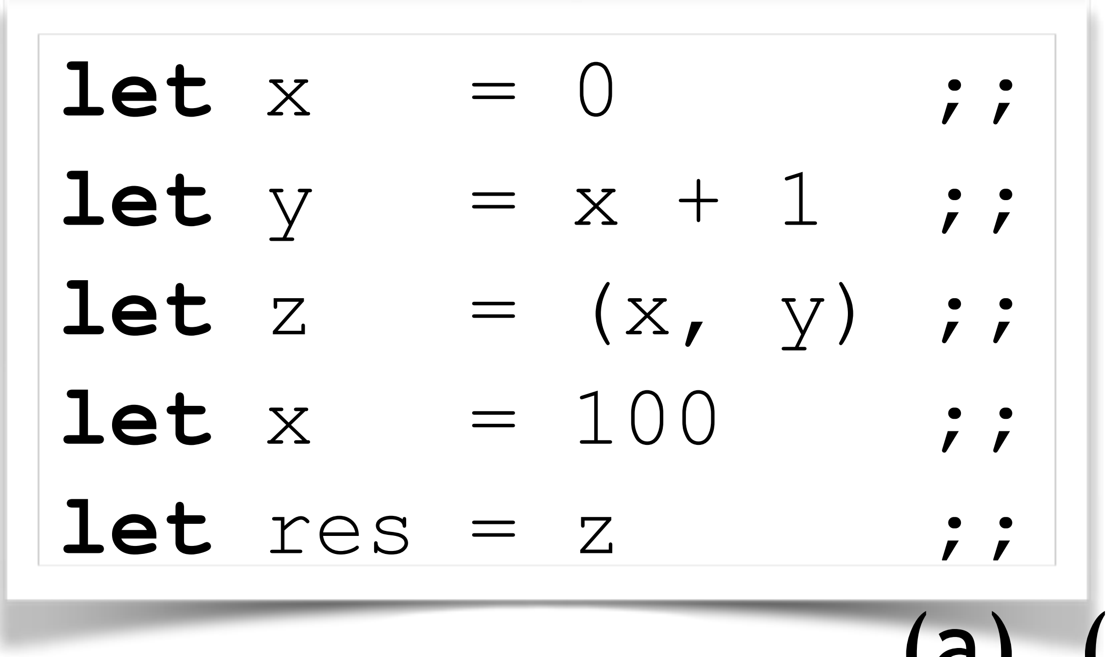
1. Evaluate:

Find and use most recent value of variable

2. Extend:

Add new binding at end of "phone book"

Q: What is the value of res ?



```
let x   = 0        ;;
let y   = x + 1    ;;
let z   = (x, y)   ;;
let x   = 100      ;;
let res = z        ;;
```

(a) `(0, 1)`

(b) `(100, 101)`

(c) `(0, 100)`

(d) `(1, 100)`

Example

```
# let x = 2+2;;
val x : int = 4

# let y = x * x * x;;
val y : int = 64

# let z = [x;y;x+y];;
val z : int list = [4;64;68]


# let x = x + x ;;
val x : int = 8
```

…	…

…	…
x	4 : int

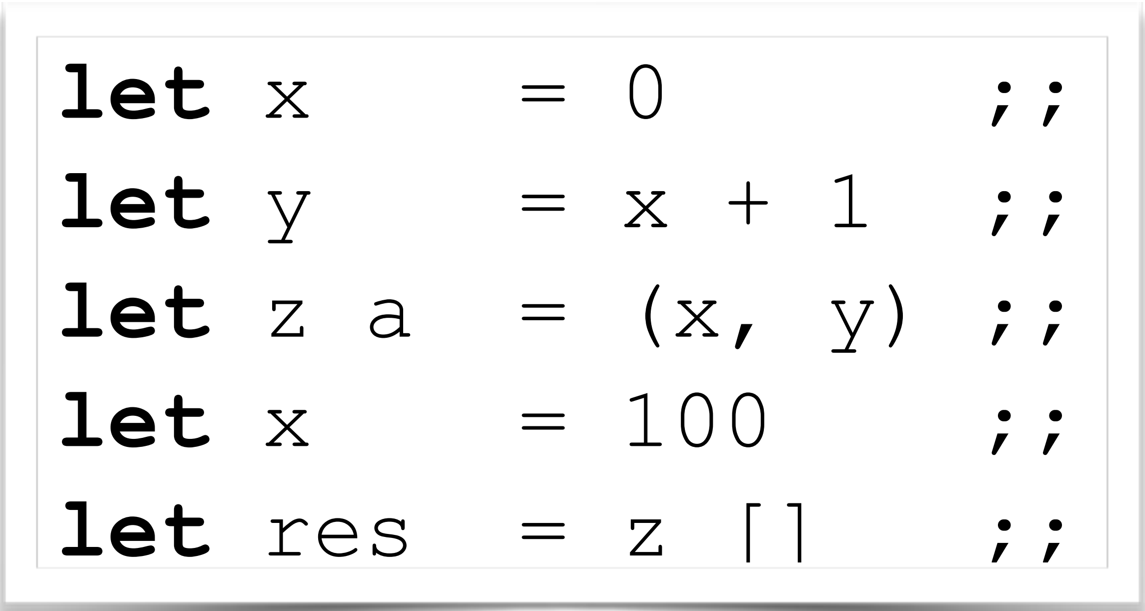
…	…
x	4 : int
y	64 : int

…	…
x	4 : int
y	64 : int
z	[4;64;68] : int list

…	…
x	4 : int
y	64 : int
z	[4;64;68] : int list
x	8 : int

New binding!

Q: What is the value of res ?



```
let x     = 0        ;;
let y     = x + 1    ;;
let z a   = (x, y)   ;;
let x     = 100      ;;
let res   = z []     ;;
```

(a) `(0, 1)`

(b) `(100,101)`

(c) `(0, 100)`

(d) `(100, 1)`

Environments

1. Evaluate: Use most recent bound value of var
2. Extend: Add new binding at end

How is it different from C/Java's "store" ?

```
# let x = 2+2;;
val x : int = 4

# let f = fun y -> x + y;
val f : int -> int = fn

# let x = x + x ;
val x : int = 8

# f 0;
val it : int = 4
```

…	…
x	*4 : int*

…	…
x	*4 : int*
f	fn <*code*, >: *int->int*

New binding:

- No change or mutation
- Old binding frozen in **f**

Environments

1. Evaluate: Use most recent bound value of var
2. Extend: Add new binding at end

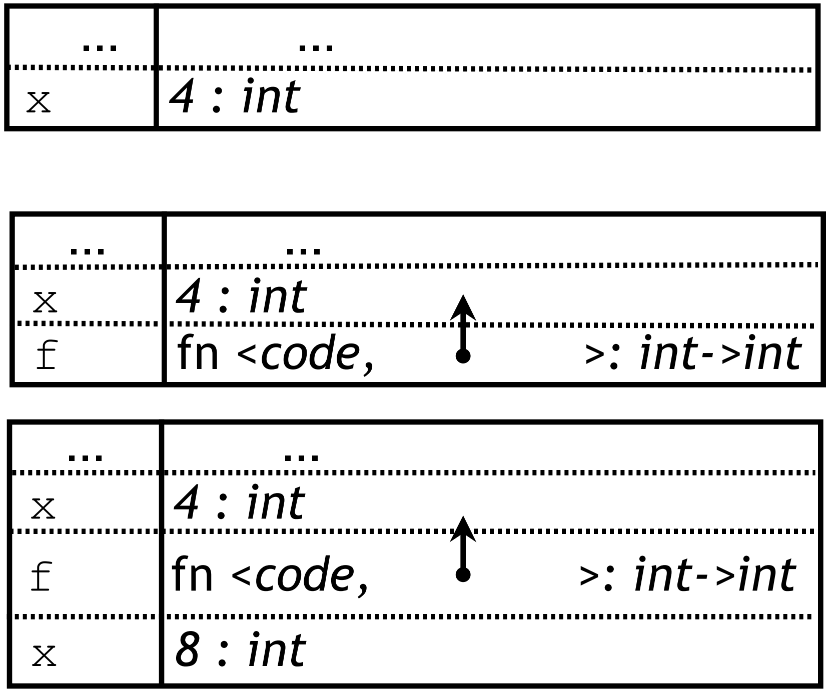
How is it different from C/Java's "store" ?

```
# let x = 2+2;
val : int x = 4

# let f = fun y -> x + y;
val f : int -> int = fn

# let x = x + x ;
val x : int = 8;

# f 0;
val it : int = 4
```



…	…
x	4 : int

…	…
x	4 : int
f	fn <code, >: int->int

…	…
x	4 : int
f	fn <code, >: int->int
x	8 : int

Environments

1. Evaluate: Use most recent bound value of var
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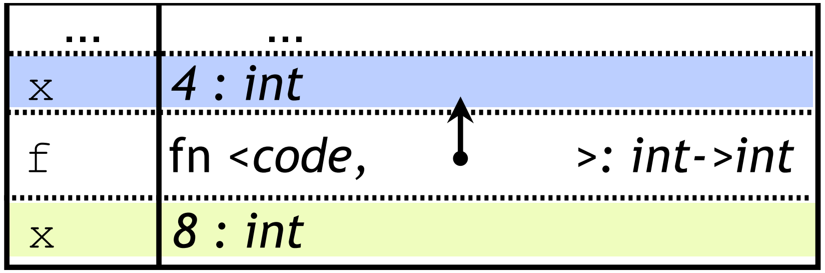
How is it different from C/Java's "store" ?

```
# let x = 2+2;
val x : int = 4

# let f = fun y -> x + y;;
val f : int -> int = fn

# let x = x + x ;
val x : int = 8

# f 0;
val it : int = 4
```

Binding used to eval (f …)



…	…
x	4 : int
f	fn <code, >: int->int
x	8 : int

Binding for subsequent x

Cannot change the world

Cannot "assign" to variables

- Can extend the env by adding a fresh binding
- Does not affect previous uses of variable

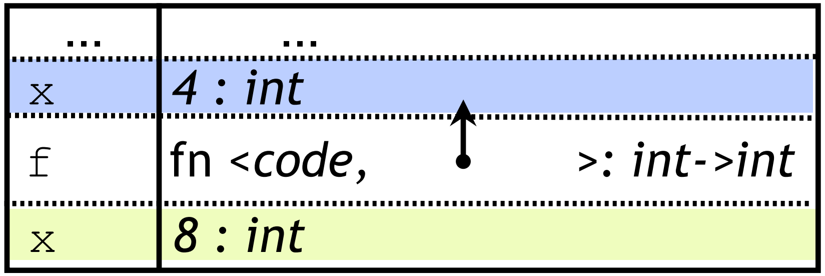
Environment at fun declaration **frozen inside fun** "value"

- Frozen env used to evaluate **application** **`(f e)`**

Q: Why is this a good thing ?

```
# let x = 2+2;;
val x : int = 4
# let f = fun y -> x + y;;
val f : int -> int = fn
# let x = x + x ;;
val x : int = 8;
# f 0;;
val it : int = 4
```

Binding used to eval `(f …)`



…	…
x	*4 : int*
f	fn <*code*, >: *int->int*
x	*8 : int*

Binding for subsequent **`x`**

Cannot change the world

Q: Why is this a good thing ?

A: Function behavior frozen at declaration

Immutability: The Colbert Principle

"A function behaves the same way on Wednesday, as it behaved on Monday, *no matter what happened on Tuesday!*"

Cannot change the world

Q: Why is this a good thing ?

A: Function behavior frozen at declaration

- Nothing entered afterwards affects function
- Same inputs always produce same outputs
  - Localizes debugging
  - Localizes reasoning about the program
  - No “sharing” means no evil aliasing

Examples of no sharing

Remember: No addresses, no sharing.

- Each variable is bound to a "fresh instance" of a value

Tuples, Lists …

- Efficient implementation without sharing ?
  - There is sharing and pointers but hidden from you

- Compiler's job is to optimize code
  - Efficiently implement these "no-sharing" semantics

- Your job is to use the simplified semantics
  - Write correct, cleaner, readable, extendable systems

Function bindings

Functions are values, can bind using **`val`**

```
let fname = fun x -> e ;;
```

Problem: Can’t define recursive functions !

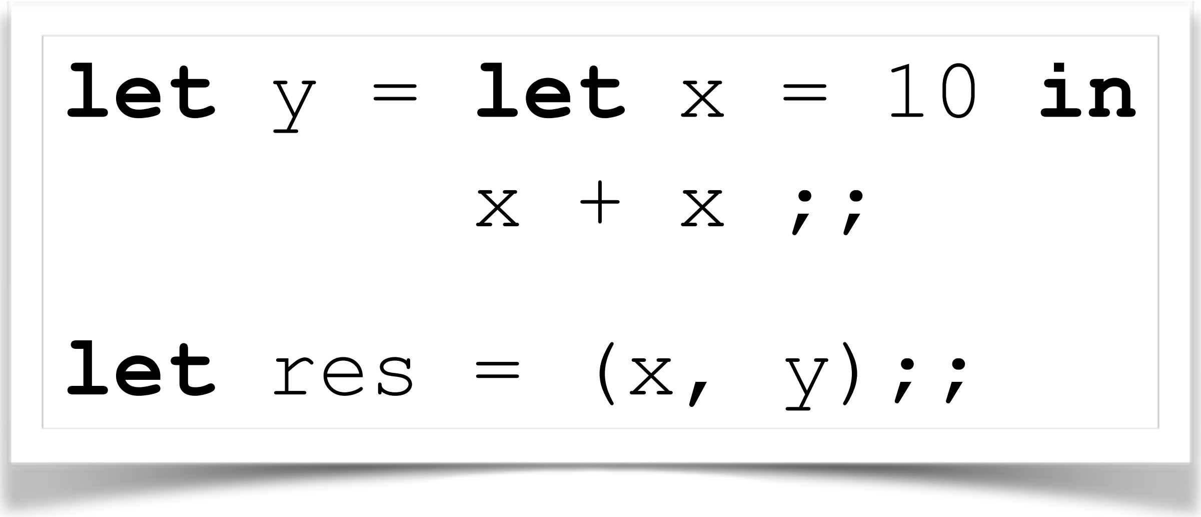
- `fname` is bound after computing rhs value
- no (or “old”) binding for occurences of `fname` inside ***e***

```
let rec fname x = e ;;
```

Occurences of `fname` inside ***e*** bound to “this” definition

```
let rec fac x = if x<=1 then 1 else x*fac (x-1)
```

Q: What is the value of res ?



```
let y = let x = 10 in
        x + x ;;

let res = (x, y);;
```

(a) Syntax Error

(b) `(10,20)`

(c) `(10,10)`

(d) Type Error

Q: What is the value of res ?

```
let f x = 1;;
let f x = if x<2 then 1 else (x * f(x-1));;
let res = f 5;;
```

(a) 120

(b) 60

(c) 20

(d) 5

(d) 1

Local bindings

So far: bindings that remain until a re-binding (“global”)

Local, “temporary” variables are useful inside functions

- Avoid repeating computations
- Make functions more readable

```
let x = e1 in
  e2
;;
```

Let-in is an expression!

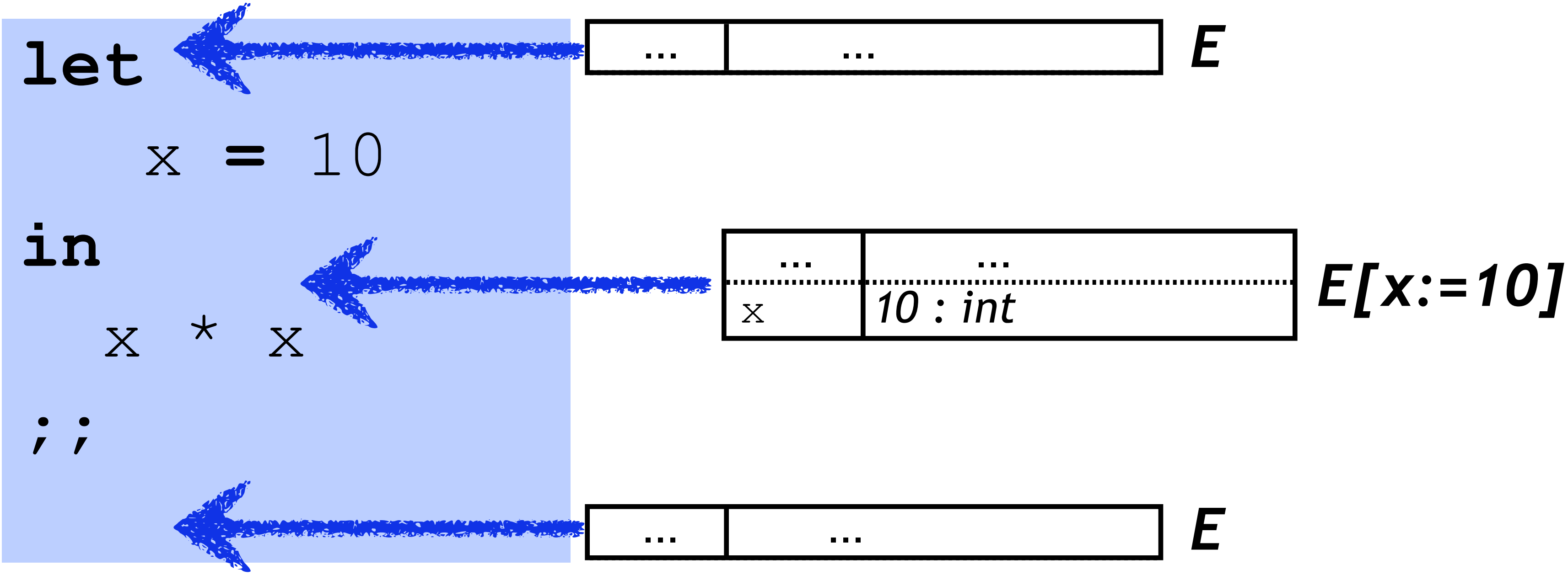
Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value *v* : ***t***
2. Use extended ***E*** [**x** |-> *v* : ***t***] (only) to evaluate ***e2***

Local bindings

Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] to evaluate ***e2***

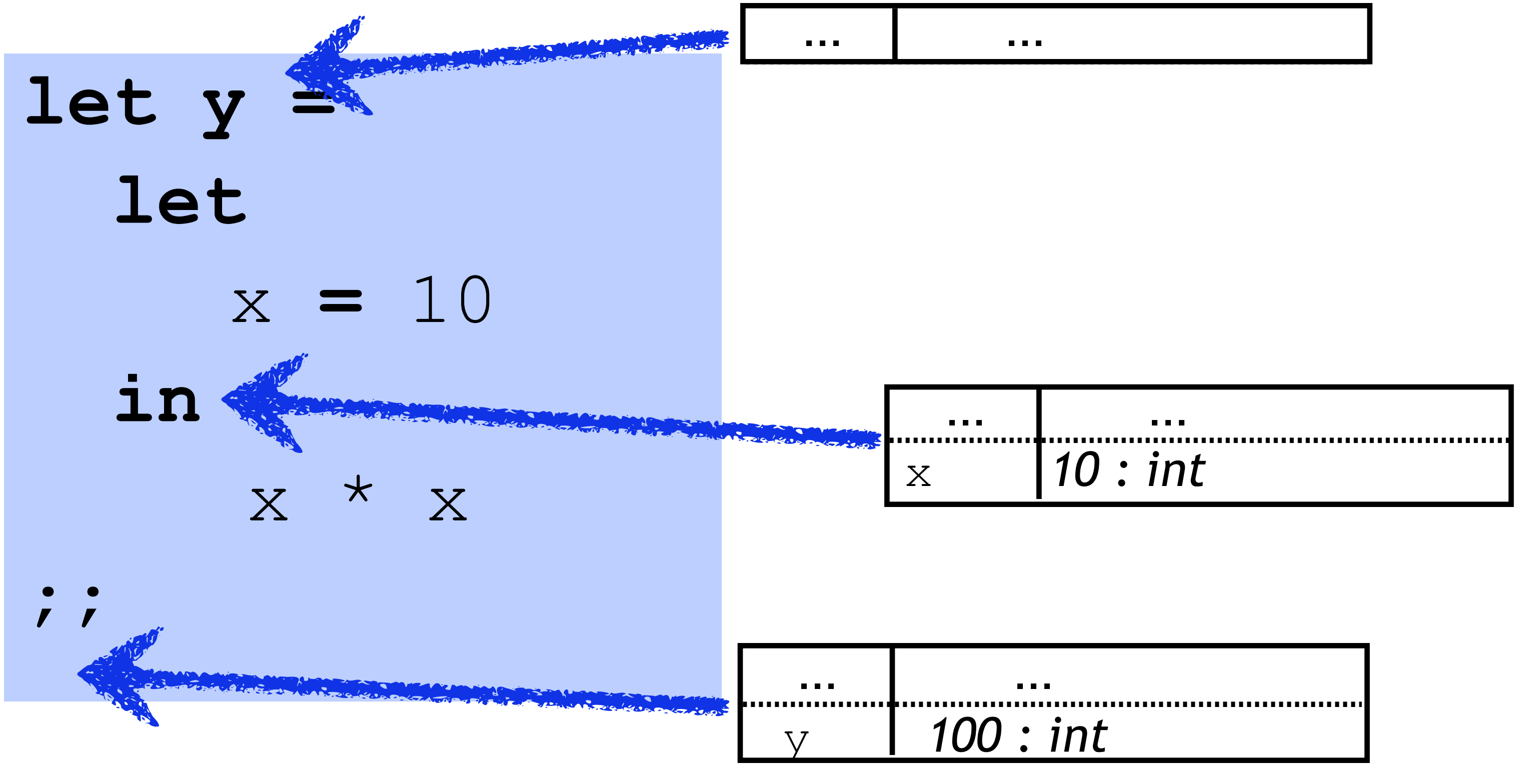


```
let
    x = 10
in
   x * x
;;
```

Let-in is an expression!

Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] to evaluate ***e2***



```
let y =
  let
     x = 10
  in
     x * x
;;
```

…	…

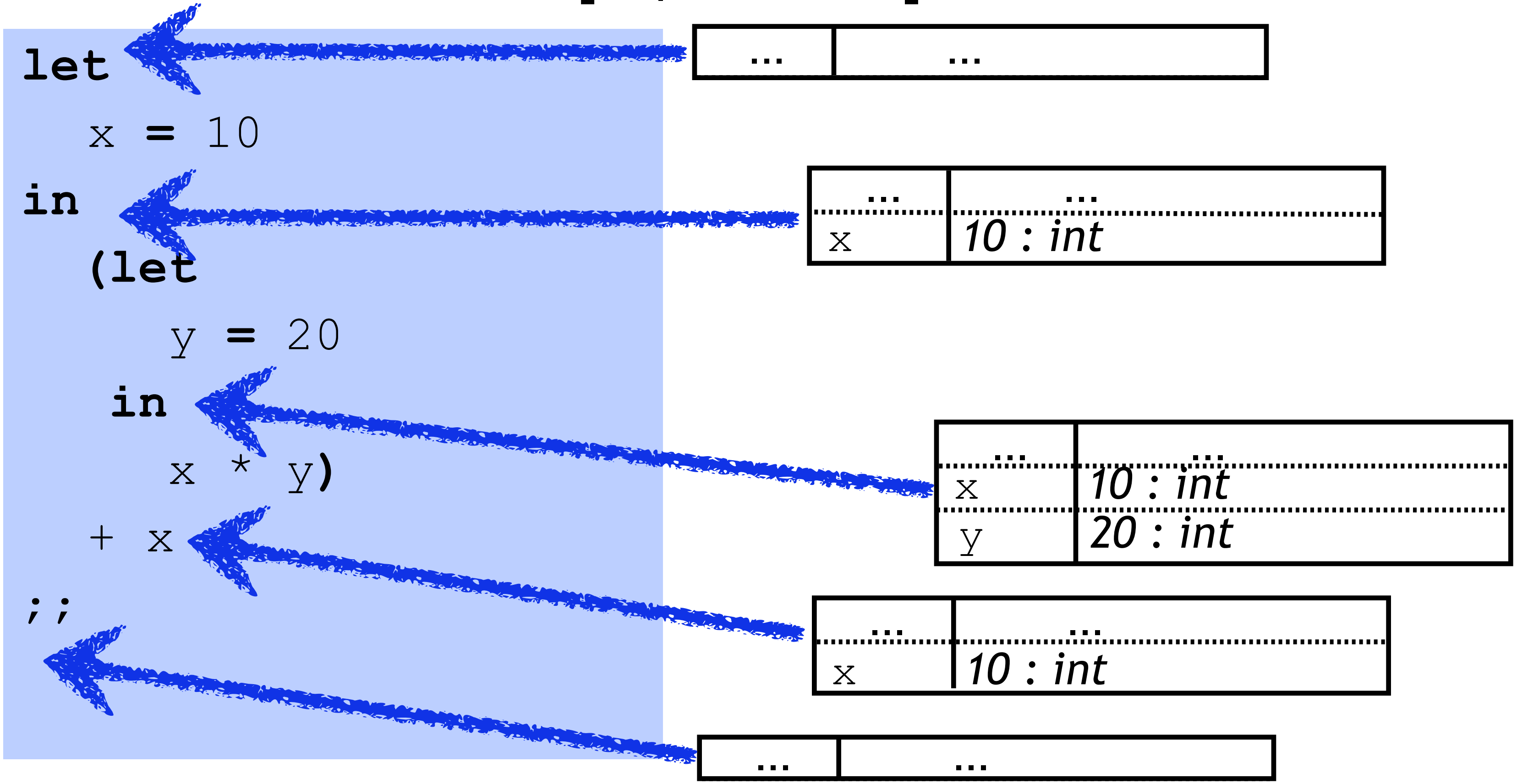
…	…
x	*10 : int*

…	…
y	*100 : int*

Nested bindings

Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] to evaluate ***e2***



```
let
   x = 10
in
   (let
      y = 20
    in
      x * y)
   + x
;;
```

Nested bindings

```
let
  x = 10
in
  let
    y = 20
  in
    x * y
;;
```

BAD Formatting

```
let x = 10 in
let y = 20 in
  x * y
;;
```

GOOD Formatting

Example

```
let rec filter f xs =
  match xs with
  | []     -> []
  | x::xs’ -> let ys  = if f x then [x] else [] in
              let ys’ = filter f xs            in
               ys @ ys’
```

Recap 1: Variables are names for values

- Environment: dictionary/phonebook
- Most recent binding used
- **Entries never change**
- New entries added

Recap 2: Big Exprs With Local Bindings

- **`let-in`** expression
- Variable “in-scope” **`in`**-expression
- Outside, variable not “in-scope”

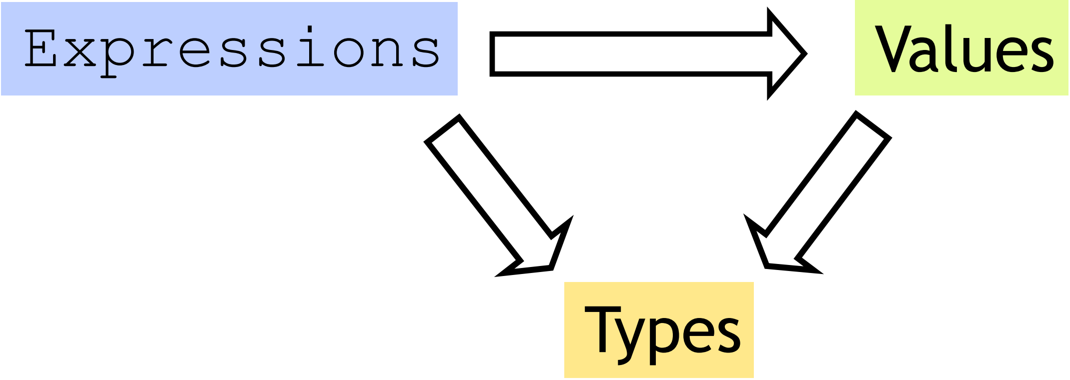
Recap 3: Env Frozen at Func Definition

- Re-binding vars cannot change function
- Indentical I/O behavior at every call
- Predictable code, localized debugging

Static/Lexical Scoping

- For each occurrence of a variable,
  A unique place where variable was defined!
  - Most recent binding in environment
- Static/Lexical: Determined from program text
  - Without executing the program
- Very useful for readability, debugging:
  - Don’t have to figure out “where” a variable got assigned
  - Unique, statically known definition for each occurrence

Next: Functions



Q: What's the value of a function ?

Immutability: The Colbert Principle

“A function behaves the same way on Wednesday, as it behaved on Monday, *no matter what happened on Tuesday!*”

Functions

Two ways of writing function expressions:

1. Anonymous functions:

Parameter (formal)

Body Expr

```
let fname = fun x -> e
```

2. Named functions:

Parameter (formal)

Body Expr

```
let fname x = e
```

Function Application `Expressions`

Application: fancy word for “call”

```
(e1 e2)
```

- Function value *e1*
- Argument *e2*
- “apply” argument *e2* to function value *e1*

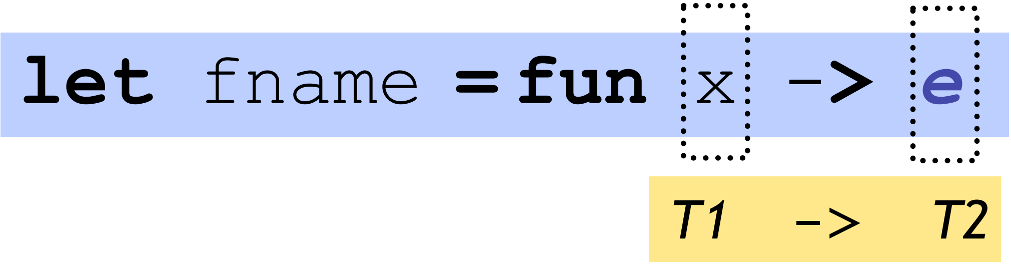
Functions

Type

The type of any function is:

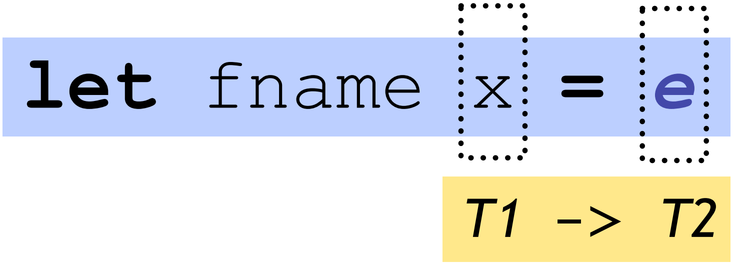
- *T1* : the type of the "input"
- *T2* : the type of the "output"

T1 -> *T2*



```
let fname = fun x -> e
```

T1 -> *T2*



```
let fname x = e
```

T1 -> *T2*

Functions

Type

The type of any function is:

- *T1* : the type of the “input”
- *T2* : the type of the “output”

T1–>T2

T1, *T2* can be any types, including functions!

Whats an example of ?

- *int –> int*
- *int * int –> bool*
- *(int –> int) –> (int –> int)*

Type of function application

Application: fancy word for “call”

```
(e1 e2)
```

- “apply” argument `e2` to function value `e1`

$$\frac{\mathtt{e1} : T1 \texttt{ -> } T2 \qquad \mathtt{e2} : T1}{\mathtt{(e1\ e2)} : T2}$$

- Argument must have same type as “input” *T1*
- Result has the same type as “output” *T2*

Functions

Values

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

2. … of a function “application” (call) ? `(e1 e2)`

Values of function = “Closure”

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

Closure =

Code of Fun. (formal `x` + body `e`)

+ Environment at Fun. Definition

Values of function = “Closure”

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

Closure =

Code of Fun. (formal **x** + body ***e***)

+ Environment at Fun. Definition

Q: Which vars in closure of f ?

```
let x   = 2 + 2 ;;
let f y = x + y ;;
let z   = x + 1 ;;
```

(a) x

(b) y

(c) x y

(d) x y z

(e) None

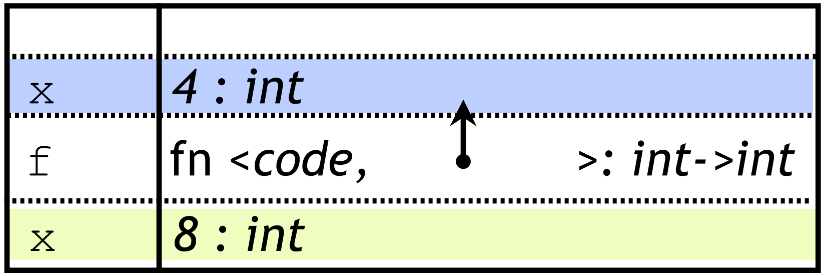
Values of functions: Closures

- Function value = **"Closure"**
  - <code + environment at definition>

- Body not evaluated until application
  - But type-checking when function is defined

```
# let x = 2+2;;
val x : int = 4
# let f = fun y -> x + y;;
val f : int -> int = fn
# let x = x + x;;
val x : int = 8
# f 0;;
val it : int = 4
```

Binding used to eval (f …)



x	4 : int
f	fn <code, >: int->int
x	8 : int

Binding for subsequent x

Q: Which vars in closure of f ?

```
let a = 20;;

let f x =
  let y   = 1 in
  let g z = y + z in
    a + (g x)
;;
```

(a) a y

(b) a

(c) y

(d) z

(e) y z

Free vs. Bound Variables

```
let a = 20;;

let f x =
  let y = 1 in
  let g z = y + z in
    a + (g x)
;;

f 0;;
```

```
(e1 e2)
```

Environment frozen with function

Used to evaluate fun application

Which vars needed in frozen env?

Free vs. Bound Variables

```
let a = 20;;

let f x =
  let y = 1 in
  let g z = y + z in
    a + (g x)
;;

f 0;;
```

Inside a function:

A "bound" occurrence:

1. Formal variable
2. Variable bound in `let-in`

`x`, `y`, `z` are "bound" inside `f`

A "free" occurrence:

- **Non-bound** occurrence

`a` is "free" inside `f`

Frozen Environment

needed for values of free vars

Q: Which vars are free in f ?

```
let a = 20;;

let f x =
  let a    = 1 in
  let g z = a + z in
    a + (g x)
;;
```

(a) a

(b) x

(c) y

(d) z

(e) None

Free vs. Bound Variables

```
let a = 20;;

let f x =
   let a = 1 in
   let g z = a + z in
      a + (g x)
   ;;

f 0;
```

Inside a function:

A “bound” occurrence:

1. Formal variable
2. Variable bound in `let-in-end`

`x`, `a`, `z` are “bound” inside `f`

A “free” occurrence:

Not bound occurrence

nothing is “free” inside `f`

Where do bound-vars values come from?

```
let a = 20;;

let f x =
  let a = 1 in
  let g z = a + z in
    a + (g x)
  ;;

f 0;
```

Bound values determined when function is evaluated (“called”)

- Arguments
- Local variable bindings

Values of function application

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

2. … of a function “application” (call) ? `(e1 e2)`

“apply” the argument *e2* to the (function) *e1*

Values of function application

Value of a function "application" (call) `(e1 e2)`

1. Find closure of *e1*
2. Execute body of closure with param *e2*

Free values found in closure-environment

Bound values by executing closure-body

Values of function application

Value of a function “application” (call) `(e1 e2)`

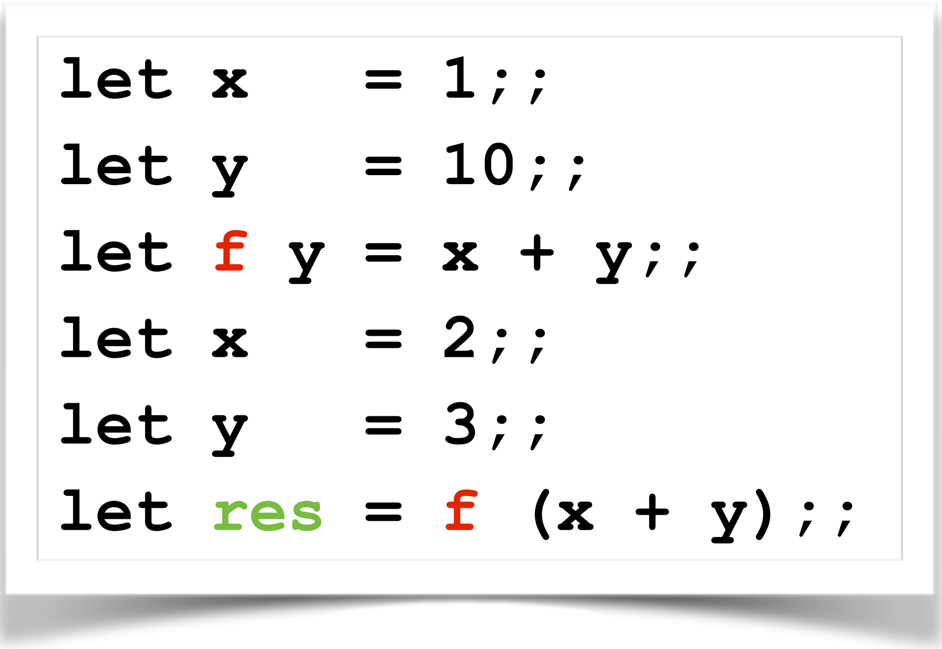
1. Evaluate `e1` in current-env to get (**closure**)

   = **code** (formal `x` + body `e`) **+ env** *E*

2. Evaluate `e2` in current-env to get (argument) *v2*

3. Evaluate body `e` in env *E* extended with `x` := *v2*

Q: What is the value of res ?



```
let x     = 1;;
let y     = 10;;
let f y = x + y;;
let x     = 2;;
let y     = 3;;
let res = f (x + y);;
```

(a) 4   (b) 5   (c) 6   (d) 11   (e) 12

Q: What is the value of res ?

```
let x   = 1;;
let y   = 10;;
let f y = x + y;;
let x   = 2;;
let y   = 3;;
let res = f (x + y);;
```

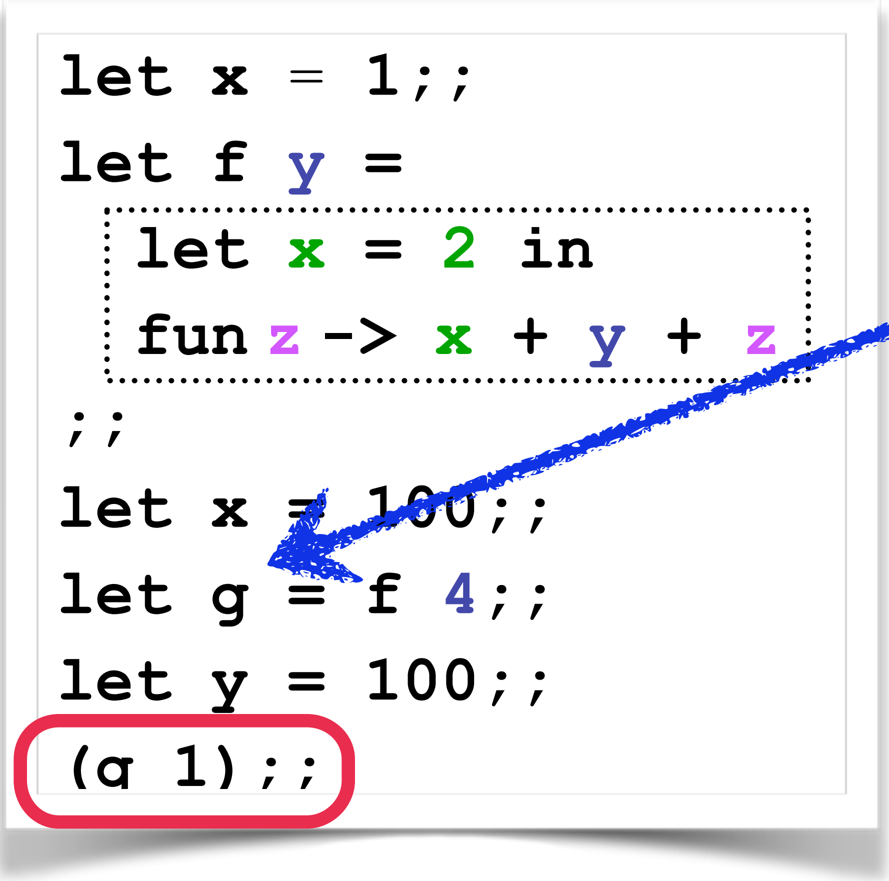
```
f |-> formal:= y
      body   := x + y
      env    := [x|->1]
x |-> 2
y |-> 3

x + y ====> 5
```

Application: `f (x + y)`

Eval `body` in `env` extended with `formal|-> 5`

Eval `x+y` in `[x|->1, y|->5] ====> 6`

Example



```
let x = 1;;
let f y =
   let x = 2 in
   fun z -> x + y + z
;;
let x = 100;;
let g = f 4;;
let y = 100;;
(g 1);;
```

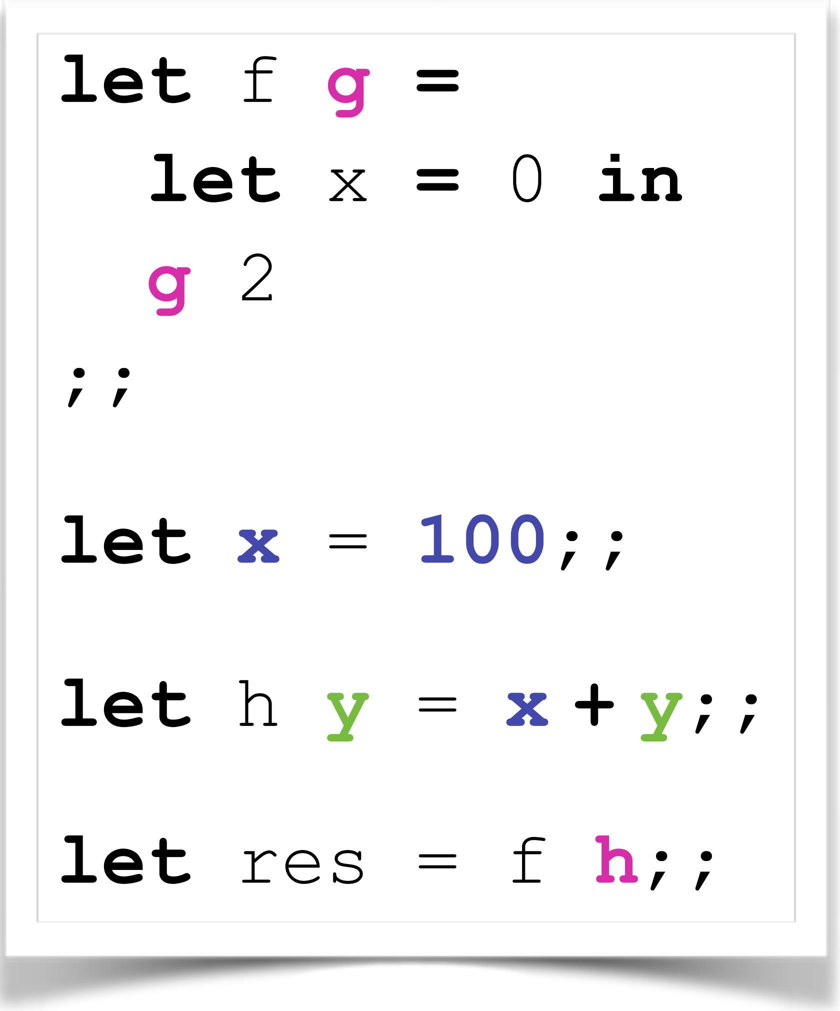
Q: Closure value of g?

```
formal  z
body     x + y + z
env      [x|->2, y|->4]
```

Eval body in env extended with formal|-> 1

Eval x+y+z in [x|->2, y|->4,z|->1] ====> 7

Q: What is the value of `res` ?



```
let f g =
  let x = 0 in
  g 2
;;

let x = 100;;

let h y = x + y;;

let res = f h;;
```

(a) Syntax Error

(b) 102

(c) Type Error

(d) 2

(e) 100

Example 3

```
let f g =
  let x = 0 in
  g 2
;;

let x = 100;;

let h y = x + y;;

f h;;
```

Static/Lexical Scoping

- For each occurrence of a variable,
  - Unique place in program text where variable defined
  - Most recent binding in environment
- Static/Lexical: Determined from the program text
  - Without executing the program
- Very useful for readability, debugging:
  - Don’t have to figure out “where” a variable got assigned
  - Unique, statically known definition for each occurrence

Immutability: The Colbert Principle

“A function behaves the same way on Wednesday, as it behaved on Monday, *no matter what happened on Tuesday!*”