CSE 20 - Spring 2014
Quiz 4

# Quiz 4a Solutions[1]

**1** Prove that for all $n \in \{1, 2, 3, \ldots\}$,

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

**Base Case**
When $n = 1$, LHS = $1^3 = 1$. RHS = $\frac{1^2 \cdot (1+1)^2}{4} = \frac{4}{4} = 1$.

**Induction Hypothesis**
Assume that for some $n$, the statement is true. That is,

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

**Proof by Induction**
Now, we have to prove that the statement is true for $n + 1$. That is, to prove:

$$1^3 + 2^3 + \ldots + n^3 + (n+1)^3 = \frac{(n+1)^2(n+2)^2}{4}$$

.

$$\begin{aligned} 1^3 + 2^3 + \ldots + (n+1)^3 &= (1^3 + 2^3 + \ldots + n^3) + (n+1)^3 \\ &= \frac{n^2(n+1)^2}{4} + (n+1)^3 \\ &= (n+1)^2[\frac{n^2}{4} + (n+1)] \\ &= \frac{1}{4}(n+1)^2[n^2 + 4(n+1)] \\ &= \frac{1}{4}(n+1)^2(n+2)^2 \end{aligned}$$

Hence, this proves that the statement is true for $n + 1$. So, by induction, we can say that the statement is true for any integer $n$. ■

[1] Questions and solution to 5b are similar to 4a

**2** Let $T(1), T(2), T(3), \ldots, T(n)$ be a sequence of numbers such that for all $n \geq 2$ then

$$T(n) = 2T(n-1) + 2^{n+1}$$

If $T(1) = 4$ then prove that for all $n \geq 1$,

$$T(n) = n2^{n+1}$$

**Base Case**
When $n = 1$, $T(1) = 1 \cdot 2^{1+1} = 4$.

**Induction Hypothesis**
Assume that for some $n$, the statement is true. That is,

$$T(n) = n2^{n+1}$$

**Proof by Induction**
Now, we have to prove that the statement is true for $n+1$. That is, to prove:

$$T(n) = (n+1)2^{n+2}$$

We know from the recurrence equation that,

$$\begin{aligned} T(n+1) &= 2T(n) + 2^{n+2} \\ &= 2n2^{n+1} + 2^{n+2} \\ &= n2^{n+2} + 2^{n+2} \\ &= (n+1) \cdot 2^{n+2} \end{aligned}$$

Hence, this proves that the statement is true for $n+1$. So, by induction, we can say that the statement is true for any integer $n$. ■