CSE 20: Assignment Set 3

1. Let $r =$ "she registered to vote" and $v =$ "she voted". Write the following statement in symbolic form: She registered to vote but she did not vote.

Solution. $r \wedge \sim v$

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2. Make a truth table for $(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$

Solution.

p	q	r	$\sim p$	$\sim p \vee q$	$p \vee (\sim p \vee q)$	$\sim r$	$q \wedge \sim r$	$\sim (q \wedge \sim r)$	$(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$
T	T	T	F	T	T	F	F	T	T
T	T	F	F	T	T	T	T	F	F
T	F	T	F	F	T	F	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	F	F	T	T
F	T	F	T	T	T	T	T	F	F
F	F	T	T	T	T	F	F	T	T
F	F	F	T	T	T	T	F	T	T

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3. Using DeMorgan's rule, state the negation of the statement: "The car is out of gas or the fuel line is plugged."

Solution. Let $p =$ "the car is out of gas" and $q =$ "the fuel line is plugged." Then, the statement $s = p \vee q$. The negation of s is $\sim s = \sim (p \vee q) = \sim p \wedge \sim q$ by DeMorgan's rule. So the negation reads "The car is not out of gas and the fuel line is not plugged."

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4. A pair of numbers x and y satisfy a system of inequalities if

$$\begin{cases} 3 \leq x \leq 5 \text{ and} \\ |x - y| < 1. \end{cases}$$

What are the conditions under which x and y fail to satisfy this system?

Solution. This system can also be written as a conjunction of two statements. Let $pbethestatement3 \leq x \leq 5$ and $qbethestatement|x-y| < 1$. Then $\sim (p \wedge q) = \sim p \vee \sim q$ by DeMorgan's rule. So system fails when $x < 3$ or $x > 5$ or $|x-1| \geq 1$

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5. Is the function $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$ equal to the function $p \vee q$? Why or why not?

Solution. No.

I will use two methods to solve this question. The first way is uses truth tables and the second way uses the algebraic rules.

Method 1

p	q	$\sim p$	$\sim p \vee q$	$\sim (\sim p \vee q)$	$p \wedge (\sim (\sim p \vee q))$	$p \wedge q$	$(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$	$p \vee q$
T	T	F	T	F	F	T	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	F	F	T
F	F	T	T	F	F	F	F	F

The last two columns of the table are not the same, and thus, the two statements are not equivalent.

Method 2

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\begin{array}{rl}
(p \wedge (\sim (\sim p \vee q)) \vee (p \wedge q) & \\
(p \wedge (\sim\sim p \wedge \sim q)) \vee (p \wedge q) & \text{DeMorgan's rule} \\
(p \wedge (p \wedge \sim q)) \vee (p \wedge q) & \text{double negation} \\
((p \wedge p) \wedge \sim q) \vee (p \wedge q) & \text{associative rule} \\
(p \wedge \sim q)) \vee (p \wedge q) & \text{idempotent} \\
p \wedge (\sim q \vee q) & \text{distributive rule} \\
p \wedge T & \text{negation} \\
p & \text{bound rule}
\end{array}
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Since p is not equivalent to $p \vee q$, neither is $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$.

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6. Prove that proving $(A \implies B)$ is the same as proving $(\neg B \implies \neg A)$.

Solution. By truth table:

A	B	$A \implies B$	$\neg B$	$\neg A$	$\neg B \implies \neg A$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

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Since the columns for $A \implies B$ and $\neg B \implies \neg A$ are the same, they are equivalent.

7. Prove that $(A \implies B)$ is equivalent to $(\neg B \wedge A) = False$.

Solution. We want to show that $(A \implies B) \iff (\neg B \wedge A)$ is always false. We can do so by constructing a truth table:

A	B	$A \implies B$	$\neg B$	$\neg B \wedge A$	$(A \implies B) \iff (\neg B \wedge A)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

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The last column proves that "$(A \implies B)$ is equivalent to $(\neg B \wedge A)$" is false.

8. Prove that if $A = X \vee Y \vee Z$, then if we want to prove $A \implies B$ then it is enough to prove that $(X \implies B) \wedge (Y \implies B) \wedge (Z \implies B)$

Solution. *Note:* This can be done using truth tables, but that would require $2^4 = 16$ rows. Instead, it is much easier to use algebraic manipulations.

We know that $A \implies B = \neg A \vee B$. Since $A = X \vee Y \vee Z$, by substitution, we get:

$$\begin{aligned} &\neg(X \vee Y \vee Z) \vee B \\ &(\neg X \wedge \neg Y \wedge \neg Z) \vee B && \text{DeMorgan's law} \\ &(\neg X \vee B) \wedge (\neg Y \vee B) \wedge (\neg Z \vee B) && \text{distribution} \\ &(X \implies B) \wedge (Y \implies B) \wedge (Z \implies B) \end{aligned}$$

Thus, the statements are equivalent.

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9. Is the statement form $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$ a tautology or a contradiction or none.

Solution. The statement is a tautology.

Method 1: Truth Table

p	q	$\neg q$	$p \wedge \neg q$	$\neg p$	$\neg p \vee (p \wedge \neg q)$	$p \wedge q$	$(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$
T	T	F	F	F	F	T	T
T	F	T	T	F	T	F	T
F	T	F	F	T	T	F	T
F	F	T	F	T	T	F	T

The last column shows that the statement is a tautology.

Method 2: Algebraic Manipulations

$$\begin{aligned} &(p \wedge q) \vee (\neg p \vee (p \wedge \neg q)) && \\ &(p \wedge q) \vee ((\neg p \vee p) \wedge (\neg p \vee \neg q)) && \text{distributive rule} \\ &(p \wedge q) \vee (\mathrm{T} \wedge (\neg p \vee \neg q)) && \text{negation rule} \\ &(p \wedge q) \vee (\neg p \vee \neg q) && \text{bound rule} \\ &(p \wedge q) \vee \neg (p \wedge q) && \text{DeMorgan's rule} \\ &\mathrm{T} && \text{negation rule} \end{aligned}$$

Thus, the statement is a tautology.

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