CSE 20: Midterm - Section B

Maximum marks: 100
Total Time: 50 minutes

9th May, 2014

Name:

PID:

1. (15 Marks) If a and b are two integers such that $(a^2 - b^2)$ is divisible by 4, then prove that either 4 divides a or 4 divides b or $(a^2 - b^2)$ is divisible by 8.

2. (20 +20 Marks) A number is *rational* if it can be written as p/q, where both p and q are integers.

   (a) Prove that $(\sqrt{5} + \sqrt{12})$ is not rational.
   (You can use the fact that $\sqrt{7}$ and $\sqrt{3}$ is not rational.)

   (b) Also prove that $\sqrt{60}$ is not rational.

3. (5 each) If A and B are two sets such that

   (a) If $|A| = 16$ and $|B| = 9$ and $|A \cap B| = 9$ then what is $|A \cup B|$.

   (b) If $|B| = 7$ what is $|B^3|$.

   (c) If $|A| = m$ how many functions are there from A to $\{-1, 0, 1\}^n$.

   (d) If $|A| = 6$ and $|B| = 8$ and $|A \cap B| = 4$ then what is the size of $|(A^c \cap B) \cup (A \cap B^c)|$.

4. (15 Marks) If P, Q and R are three propositions depending on variables x and y then what is the negation of the following:

$$\forall x \exists y \, (P(x, y) \implies (Q(x, y) \vee R(x, y)))$$

5. (15 + 10 marks)

   (a) Is the statement $(p \wedge (\neg (\neg p \vee q))) \vee (p \wedge q)$ equivalent to the statement $p \vee q$? Why or why not? Prove it using algebra (do not use truth table).

   (b) Give the truth table of the expression $(p \wedge (\neg (\neg p \vee q))) \vee (p \wedge q)$.