CSE 20

Final Review

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- Representation of integers in base b
- Logic
- Proof systems:
  - Direct Proof
  - Proof by contradiction
  - Contraposetive
- Sets Theory
- Functions
- Induction

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NO CALCULATOR, NO CHEAT SHEET

Anything that you can assume in the proofs will be clearly given.

Propositional Logic

- Every statement is either TRUE or FALSE
- There are logical connectives $\vee$, $\wedge$, $\neg$, $\implies$ and $\iff$.
- Two logical statements can be equivalent if the two statements answer exactly in the same way on every input.
- To check whether two logical statements are equivalent one can do one of the following:
  - Checking the Truthtable of each statement
  - Reducing one to the other using reductions

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- Make deductions
- Check if two propositions are equivalent.

Equivalence of statements/propositions

If the truthtables of two statement/propositions are identical then the two statement/propositions are equivalent

One can also use various rules for propositional logic.

Rules of Propositional Logic

1. Commutative law:

$$(p \vee q) = (q \vee p) \text{ and } (p \wedge q) = (q \wedge p)$$

2. Associative law:

$$(p \vee (q \vee r)) = ((p \vee q) \vee r) \text{ and}$$
$$(p \wedge (q \wedge r)) = ((p \wedge q) \wedge r)$$

3. Distributive law:

$$(p \vee (q \wedge r)) = (p \vee q) \wedge (p \vee r) \text{ and}$$
$$(p \wedge (q \vee r)) = (p \wedge q) \vee (p \wedge r)$$

4. De Morgan's Law:

$$\neg(p \vee q) = (\neg p \wedge \neg q) \text{ and } \neg(p \wedge q) = (\neg p \vee \neg q)$$

Sets

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  - Set of names of all students

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  - Set of names of all students
  - Set of letters in the english alphabet
  - Set of digits. $\{0, 1, \ldots, 9\}$ or $\{0, 1\}$

Size of a set is the number of elements in the set.
Size of set A is denoted by $|A|$.

Operations on Sets

- Union, $\cup$.
  $A \cup B$ is the set of all elements that are in A OR B.
- Intesection, $\cap$
  $A \cap B$ is the set of all elements that are in A AND B.
- Complement, A^c or $\overline{A}$
  A^c is the set of elements NOT in A.
- Cartesan Product. For example: $A^3 = A \times A \times A$.

Rules of Set Theory

Let p, q and r be sets.

1. Commutative law:
$$(p \cup q) = (q \cup p) \text{ and } (p \cap q) = (q \cap p)$$
2. Associative law:
$$(p \cup (q \cup r)) = ((p \cup q) \cup r) \text{ and}$$
$$(p \cap (q \cap r)) = ((p \cap q) \cap r)$$
3. Distributive law:
$$(p \cup (q \cap r)) = (p \cup q) \cap (p \cup r) \text{ and}$$
$$(p \cap (q \cup r)) = (p \cap q) \cup (p \cap r)$$
4. De Morgan's Law:
$$(p \cup q)^c = (p^c \cap q^c) \text{ and } (p \cap q)^c = (p^c \cup q^c)$$

Set Theory and Propositional Logic

Set Theory and Propositional Logic is two mathematical language which follow very similar rules.

Predicate Logic

- There are two important symbols: $\forall$ and $\exists$.
- Some statements can be defined using a variable.
- For example: $P_x =$ "$4x^2 + 3$ is divisible by 5"
- We can have statements like: $\forall x \in \mathbb{Z}$, $4x^2 + 3$ is divisible by 5.
- Or $\exists x \in \mathbb{Z}$, $4x^2 + 3$ is divisible by 5.

Rules of negation

- $\neg(\forall x,\ P_x) = (\exists x,\ \neg P_x)$.
- $\neg(\exists x,\ P_x) = (\forall x,\ \neg P_x)$.

Proof Techniques

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- Direct Proof:

$$A \implies B$$

- Proof by contradiction:

$$(\neg B \wedge A) \text{ gives a contradiction}$$

- Proof by Contraposetive: $A \implies B$ is same as proving $\neg B \implies \neg A$.

- Induction

Proofs done in class

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- If $a_n = a_{n-1}+2$ and $a_1 = 2$ prove that $a_n = 2n$
- If $a_n = a_{n-1}+a_{n-2}$ and $a_1 = a_2 = 1$ then

$$a_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

Induction: Version 1

If we have to prove "$\forall n \geq r \; P(n)$ is True."

Let us induct on n.

- Base Case: Prove that the $P(r)$ is true.
- Induction Hypothesis: Let for some $k \geq r \; P(k)$ is true.
- Inductive Step: We want to show, assuming IH, $P(k+1)$ is true.

By induction we have proved that $\forall n \geq r \; P(n)$ is true.

Induction: Version 2

If we have to prove "$\forall n \geq r\ P(n)$ is True."

Let us induct on n.

- Base Case: Prove that the $P(r)$ and $P(r+1)$ is true.
- Induction Hypothesis: Let for some $k \geq r\ P(k)$ and $P(k+1)$ is true.
- Inductive Step: We want to show, assuming IH, $P(k+2)$ is true.

By induction we have proved that $\forall n \geq r\ P(n)$ is true.