CSE 20

Lecture 14: Logic and Proof Techniques

Midterm Review

- Representation of integers in base b
- Logic
- Proof systems:
  - Direct Proof
  - Proof by contradiction
  - Contraposetive
- Sets Theory
- Functions

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NO CALCULATOR, NO CHEAT SHEET

Use of Propositional Logic

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- Check the correctness of a statement: whether a sentence/paragraph/proposition is logically correct.
- Make deductions
- Check if two propositions are equivalent.

Equivalence of statements/propositions

If the truthtables of two statement/propositions are identical then the two statement/propositions are equivalent

One can also use various rules for propositional logic.

Rules of Propositional Logic

1. Commutative law:

$$(p \vee q) = (q \vee p) \text{ and } (p \wedge q) = (q \wedge p)$$

2. Associative law:

$$(p \vee (q \vee r)) = ((p \vee q) \vee r) \text{ and}$$
$$(p \wedge (q \wedge r)) = ((p \wedge q) \wedge r)$$

3. Distributive law:

$$(p \vee (q \wedge r)) = (p \vee q) \wedge (p \vee r) \text{ and}$$
$$(p \wedge (q \vee r)) = (p \wedge q) \vee (p \wedge r)$$

4. De Morgan's Law:

$$\neg(p \vee q) = (\neg p \wedge \neg q) \text{ and } \neg(p \wedge q) = (\neg p \vee \neg q)$$

Equivalence of Statement: iclicker Question 1

Which of the following is equivalent to $A \implies B$

A. $(\neg B \vee A) = False$

B. $(\neg A \wedge B) = False$

C. $(\neg B \wedge A) = False$

D. $\neg A \implies \neg B$

E. $(\neg B \vee A)$

Proof Techniques

To prove statement B from A:

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To prove statement B from A:

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- Proof by Contraposetive: $A \implies B$ is same as proving $\neg B \implies \neg A$.

Sets

- Sets

  For example:

  - Set of names of all students

Sets

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  For example:

  - Set of names of all students
  - Set of letters in the english alphabet

Sets

- Sets

  For example:

  - Set of names of all students
  - Set of letters in the english alphabet
  - Set of digits. $\{0, 1, \ldots, 9\}$ or $\{0, 1\}$

Size of a set is the number of elements in the set.
Size of set A is denoted by $|A|$.

Operations on Sets

- Union, $\cup$.
  $A \cup B$ is the set of all elements that are in A OR B.
- Intesection, $\cap$
  $A \cap B$ is the set of all elements that are in A AND B.
- Complement, A^c or $\overline{A}$
  A^c is the set of elements NOT in A.
- Cartesan Product.

Rules of Set Theory

Let p, q and r be sets.

1. Commutative law:

$$(p \cup q) = (q \cup p) \text{ and } (p \cap q) = (q \cap p)$$

2. Associative law:

$$(p \cup (q \cup r)) = ((p \cup q) \cup r) \text{ and}$$
$$(p \cap (q \cap r)) = ((p \cap q) \cap r)$$

3. Distributive law:

$$(p \cup (q \cap r)) = (p \cup q) \cap (p \cup r) \text{ and}$$
$$(p \cap (q \cup r)) = (p \cap q) \cup (p \cap r)$$

4. De Morgan's Law:

$$(p \cup q)^c = (p^c \cap q^c) \text{ and } (p \cap q)^c = (p^c \cup q^c)$$

Set Theory and Propositional Logic

Set Theory and Propositional Logic is two mathematical language which follow very similar rules.

iclicker Question 2

If A and B are two sets such that size of A is 10 and size of B is 12. If size of $A \cup B$ is 20 what is size of $A \cap B$.

- A 2
- B 3
- C 10
- D 22
- E Can't be said.

iclicker Question 3

If A and B are two sets such that size of A is 10 and size of B is 12. If size of $A \cap B$ is 4 what is size of $A \backslash B$.

A 2

B 3

C 6

D 10

E Can't be said.

iclicker Question 4

If A and B are two sets such that size of A is 10 and size of B is 12. How many functions are there from A to B.

- A 10^{12}
- B 12^{10}
- C 120
- D 12
- E None of the above.

Propositional Logic

- Every statement is either TRUE or FALSE
- There are logical connectives $\vee$, $\wedge$, $\neg$, $\implies$ and $\iff$.
- Two logical statements can be equivalent if the two statements answer exactly in the same way on every input.
- To check whether two logical statements are equivalent one can do one of the following:
  - Checking the Truthtable of each statement
  - Reducing one to the other using reductions

Predicate Logic

- There are two important symbols: $\forall$ and $\exists$.
- Some statements can be defined using a variable.
- For example: $P_x =$ "$4x^2 + 3$ is divisible by 5"
- We can have statements like: $\forall x \in \mathbb{Z}$, $4x^2 + 3$ is divisible by 5.
- Or $\exists x \in \mathbb{Z}$, $4x^2 + 3$ is divisible by 5.

Rules of negation

- $\neg(\forall x,\ P_x) = (\exists x,\ \neg P_x)$.
- $\neg(\exists x,\ P_x) = (\forall x,\ \neg P_x)$.

Negating a sentence: iclicker

What is the negation of the sentence: "There is an university in USA where every department has at least 20 faculty and at least one noble laureate."

Negating a sentence: iclicker

What is the negation of the sentence: "There is an university in USA where every department has at least 20 faculty and at least one noble laureate."

- There is an university in USA where every department has less than 20 faculty and at least one noble laureate.
- All universitis in USA where every department has at least 20 faculty and at least one noble laureate.
- For all universities in USA there is a department has less than 20 faculty or at most one noble laureate.
- For all universities in USA there is a department has less than 20 faculty and at least one noble laureate.

Is $\sqrt{2}$ a rational?

In the last class we used proof by contradiction to prove that $\sqrt{2}$ is not rational.

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Is $\sqrt{2} + \sqrt{3}$ a rational?

Prove that $\sqrt{2} + \sqrt{3}$ is not rational.