CSE 20

Lecture 12: Propositional Logic (contd...)

Use of Propositional Logic

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- Check if two propositions are equivalent.

Equivalence of statements/propositions

If the truthtables of two statement/propositions are identical then the two statement/propositions are equivalent

Equivalence of Statement: iclicker Question 1

Which of the following is equivalent to $A \implies B$

A. $\neg B \implies \neg A$

B. $(\neg A \wedge B) = False$

C. $(\neg B \wedge A) = False$

D. $\neg A \implies \neg B$

E. $(\neg B \vee A)$

Proof Techniques

To prove statement B from A:

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- Proof by Contraposetive: $A \implies B$ is same as proving $\neg B \implies \neg A$.

Equivalence of Statement: iclicker Question 2

p	q	r	f
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
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T	T	F	F
T	T	T	T

A Boolean expression f having this truth table is:

A. $[(\neg p \wedge \neg q) \vee q] \vee r$

B. $[(\neg p \wedge \neg q) \wedge q] \wedge r$

C. $[(\neg p \wedge \neg q) \wedge \neg q] \wedge r$

D. $[(\neg p \wedge \neg q) \vee q] \wedge r$

E. $[(\neg p \vee \neg q) \wedge q] \wedge r$

Equivalence of Statement: iclicker Question 3

The function

$$\Big(\ (p \vee (r \vee q)) \ \wedge \neg (p \wedge (\neg q \wedge \neg r)) \Big)$$

is equal to the function:

A. $q \vee r$

B. $\neg p \vee (r \wedge q)$

C. $(p \vee q) \vee r$

D. $(p \vee q) \wedge \neg (p \vee r)$

E. $(p \wedge r) \vee (p \wedge q)$

Rules of Propositional Logic

1. Commutative law:

$$(p \vee q) = (q \vee p) \text{ and } (p \wedge q) = (q \wedge p)$$

2. Associative law:

$$(p \vee (q \vee r)) = ((p \vee q) \vee r) \text{ and}$$
$$(p \wedge (q \wedge r)) = ((p \wedge q) \wedge r)$$

3. Distributive law:

$$(p \vee (q \wedge r)) = (p \vee q) \wedge (p \vee r) \text{ and}$$
$$(p \wedge (q \vee r)) = (p \wedge q) \vee (p \wedge r)$$

4. De Morgan's Law:

$$\neg(p \vee q) = (\neg p \wedge \neg q) \text{ and } \neg(p \wedge q) = (\neg p \vee \neg q)$$

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is equal to the function:

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B. $\neg p \vee (r \wedge q)$

C. $(p \vee q) \vee r$

D. $(p \vee q) \wedge \neg(p \vee r)$

E. $(p \wedge r) \vee (p \wedge q)$

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Let p, q and r be propositions.

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Size of a set is the number of elements in the set.

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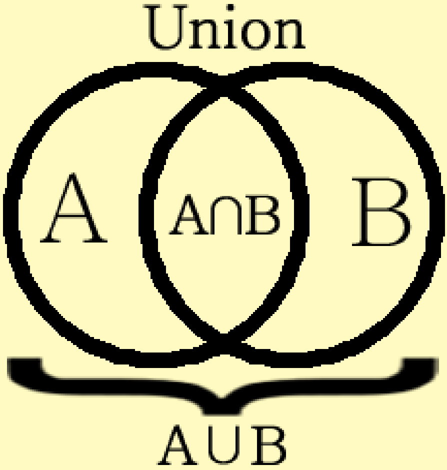
  A^c is the set of elements NOT in A.
- Cartesan Product.

Union and Intersection



Rules of Propositional Logic

Let p, q and r be propositions.

1. Commutative law:
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$$(p \vee (q \vee r)) = ((p \vee q) \vee r) \text{ and}$$
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4. De Morgan's Law:
$$\neg(p \vee q) = (\neg p \wedge \neg q) \text{ and } \neg(p \wedge q) = (\neg p \vee \neg q)$$

Rules of Set Theory

Let p, q and r be sets.

1. Commutative law:

$$(p \cup q) = (q \cup p) \text{ and } (p \cap q) = (q \cap p)$$

2. Associative law:

$$(p \cup (q \cup r)) = ((p \cup q) \cup r) \text{ and}$$
$$(p \cap (q \cap r)) = ((p \cap q) \cap r)$$

3. Distributive law:

$$(p \cup (q \cap r)) = (p \cup q) \cap (p \cup r) \text{ and}$$
$$(p \cap (q \cup r)) = (p \cap q) \cup (p \cap r)$$

4. De Morgan's Law:

$$(p \cup q)^c = (p^c \cap q^c) \text{ and } (p \cap q)^c = (p^c \cup q^c)$$

Set Theory and Propositional Logic

Set Theory and Propositional Logic is two mathematical language which follow very similar rules.

iclicker Question 4

If A and B are two sets such that size of A is 10 and size of B is 12. If size of $A \cup B$ is 20 what is size of $A \cap B$.

A 2

B 3

c 10

D 22

E Can't be said.