CSE 20: Assignment Set 4

1. Is the statement form $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$ a tautology or a contradiction or none.
2. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false. Prove the statements that are true.
   (a) $\forall x \in D$, if x is odd then $x > 0$.
   (b) $\forall x \in D$, if x is less than 0 then x is even.
   (c) $\forall x \in D$, if x is even then $x \leq 0$.
   (d) $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.
3. Write a negation for each statement. Bring the negation as deeply into the statement as possible.
   (a) For all real numbers x, if $x^2 \geq 1$ then $x > 0$.
   (b) For all integers d, if $\frac{6}{d}$ is an integer then $d = 3$.
   (c) For all real numbers x, if $x(x + 1) > 0$ then $x > 0$ or $x < -1$.
   (d) For all integers a, b, and c, if $a - b$ is even and $b - c$ is even then $a - c$ is even.
4. Let x be an integer. Prove that if $x^2 - 6x + 5$ is even then x must be odd.
5. If A and B are two sets such that $|A| = 8$ and $|B| = 9$ then can $|A \cap B|$ be equal to 10. Explain your answer.
6. If A and B are two sets such that $|A| = 8$ and $|B| = 9$ and $|A \cup B| = 15$ then what is $|A \cap B$.
7. If $S = \{0, 1\}$ and $T = \{a, e, i, o, u\}$ then what is the set $S \times T$. What is the size of the set $S \times T$.
8. How many functions are there from $\{0, 1\}^n$ to $\{1, 2, \ldots, n\}$.
9. Prove that $\sqrt{3}$ is not rational.
10. Prove that for any $n \in \mathbb{Z}^+$, $\sqrt{n} + \sqrt{2}$ is not rational.
11. Check the truth of the following statements. First convert to logic statements:
    (a) Every even integer is divisible by 4 if and only if either 7 divides 21 or 9 divides 12.

(b) Either snow is hot or 2 is even implies 3 is even

(c) $\forall x \in \mathbb{Z}$, $3x^2 + 2x + 3 \neg 0$ implies $\exists x \in \mathbb{Z}$ such that $3x^2 + 2x + 3 = 0$.

12. Write the following expression using $\neg$, $\vee$ and $\wedge$ only.

(a) $p \implies q)$

(b) $p \iff q$

(c) $[\neg p \wedge (p \vee q)] \implies q$