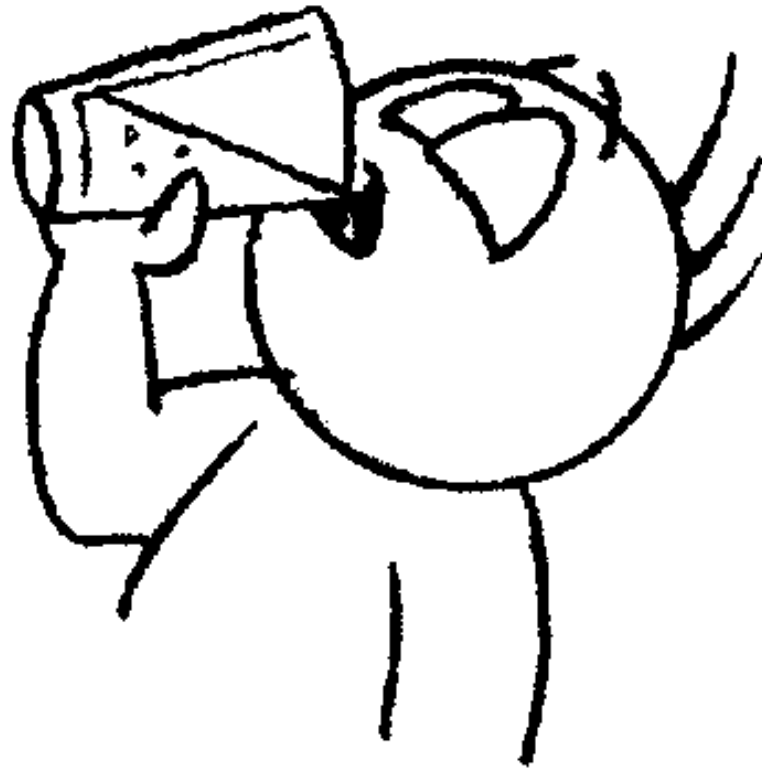


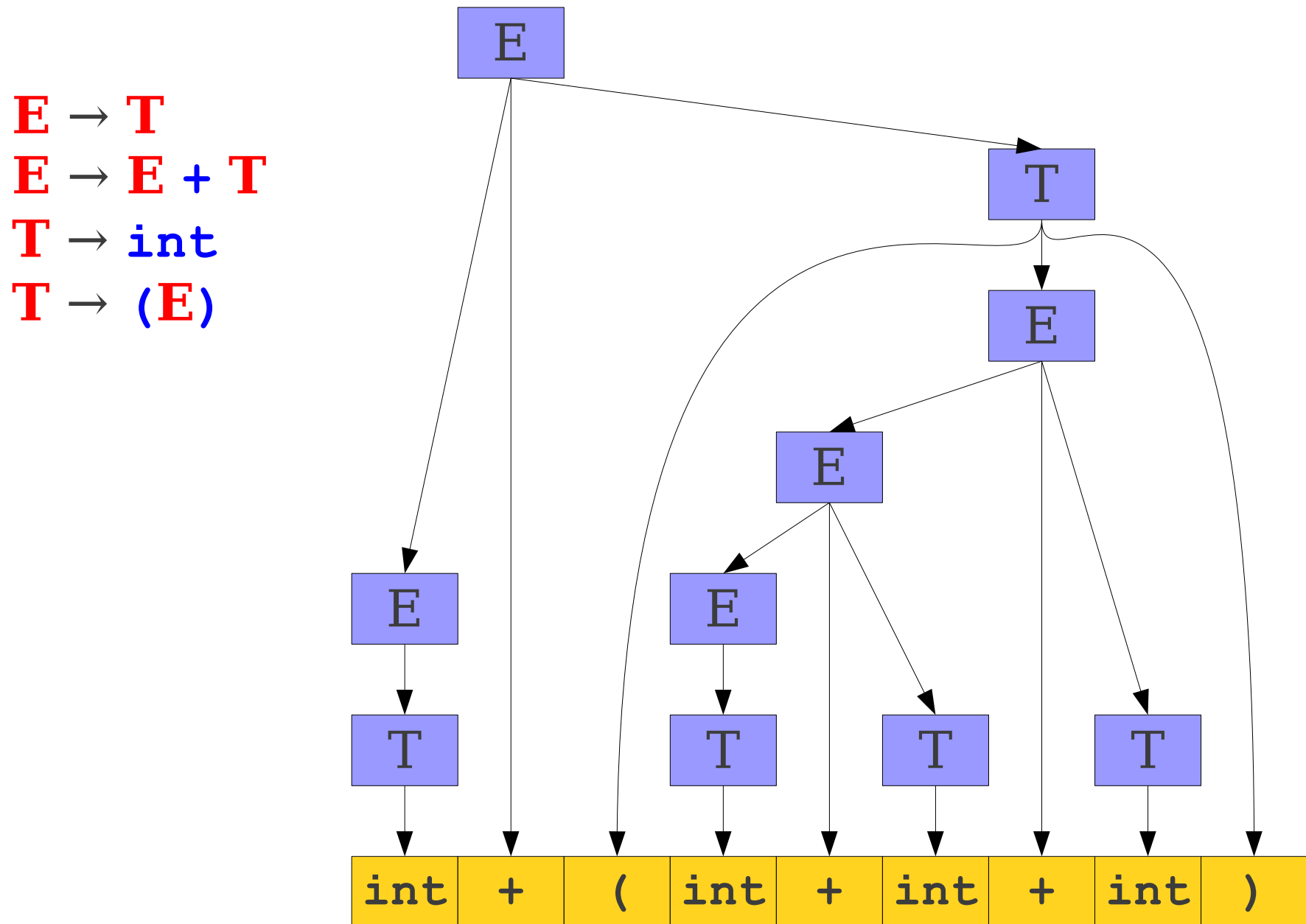
Different Types of Parsing

- Top Down Parsing
 - Beginning with the start symbol, try to guess the productions to apply to end up at the user's program.
- Bottom-Up Parsing
 - Beginning with the user's program, try to apply productions in reverse to convert the program back into the start symbol.

Bottoms Up!



One View of a Bottom-Up Parse



A Second View of a Bottom-Up Parse

$E \rightarrow T$	$\text{int} + (\text{int} + \text{int} + \text{int})$
$E \rightarrow E + T$	$\Rightarrow T + (\text{int} + \text{int} + \text{int})$
$T \rightarrow \text{int}$	$\Rightarrow E + (\text{int} + \text{int} + \text{int})$
$T \rightarrow (E)$	$\Rightarrow E + (T + \text{int} + \text{int})$
	$\Rightarrow E + (E + \text{int} + \text{int})$
	$\Rightarrow E + (E + T + \text{int})$
	$\Rightarrow E + (E + \text{int})$
	$\Rightarrow E + (E + T)$
	$\Rightarrow E + (E)$
	$\Rightarrow E + T$
	$\Rightarrow E$

A Second View of a Bottom-Up Parse

$E \rightarrow T$	$\text{int} + (\text{int} + \text{int} + \text{int})$
$E \rightarrow E + T$	$\Rightarrow T + (\text{int} + \text{int} + \text{int})$
$T \rightarrow \text{int}$	$\Rightarrow E + (\text{int} + \text{int} + \text{int})$
$T \rightarrow (E)$	$\Rightarrow E + (T + \text{int} + \text{int})$
	$\Rightarrow E + (E + \text{int} + \text{int})$
	$\Rightarrow E + (E + T + \text{int})$
	$\Rightarrow E + (E + \text{int})$
	$\Rightarrow E + (E + T)$
	$\Rightarrow E + (E)$
	$\Rightarrow E + T$
	$\Rightarrow E$

A left-to-right, bottom-up parse is a rightmost derivation traced in reverse.

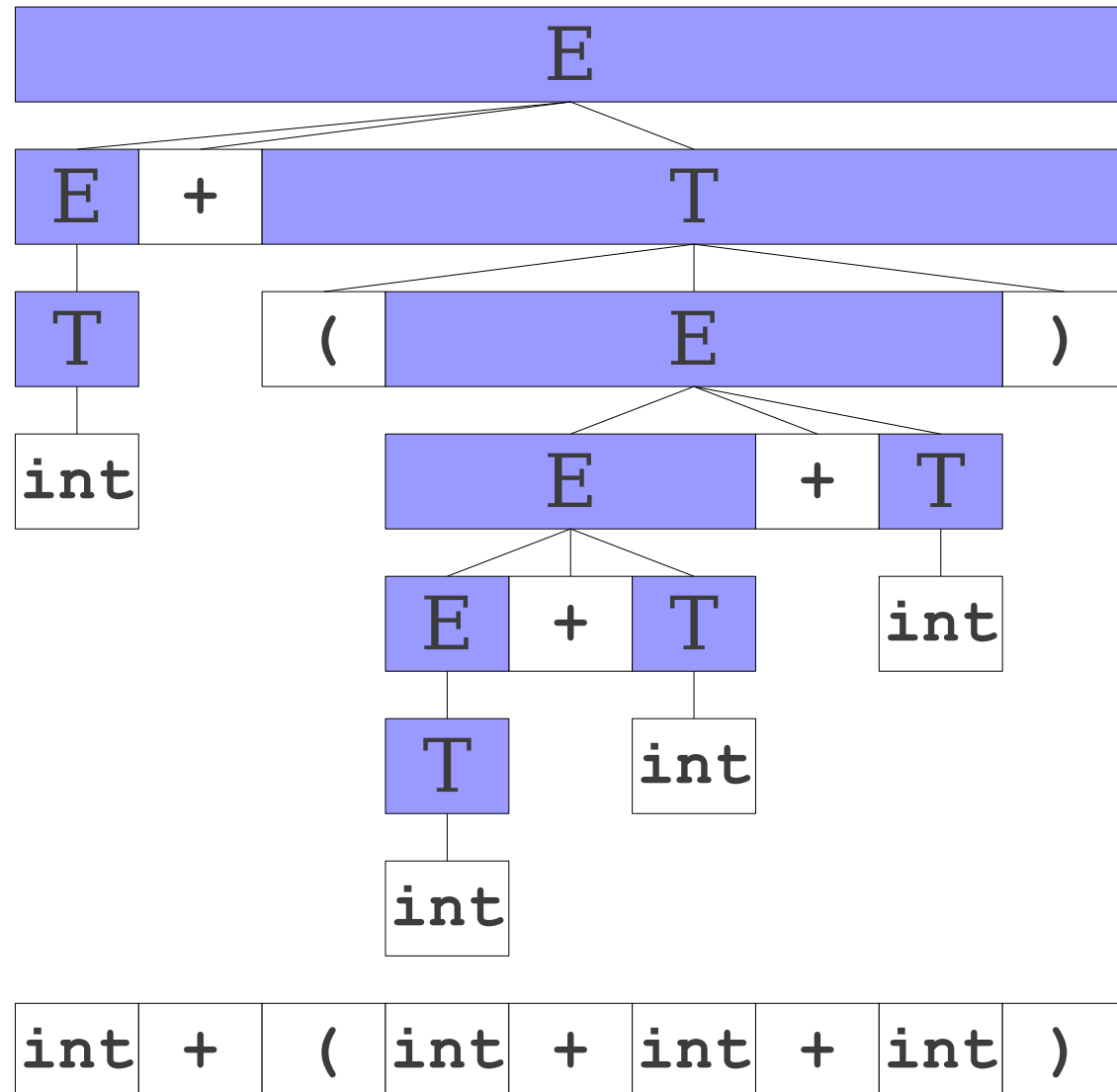
A Third View of a Bottom-Up Parse

`int + (int + int + int)`
⇒ `T + (int + int + int)`
⇒ `E + (int + int + int)`
⇒ `E + (T + int + int)`
⇒ `E + (E + int + int)`
⇒ `E + (E + T + int)`
⇒ `E + (E + int)`
⇒ `E + (E + T)`
⇒ `E + (E)`
⇒ `E + T`
⇒ `E`

Each step in this bottom-up parse is called a **reduction**.
We **reduce** a substring of the sentential form back to a nonterminal.

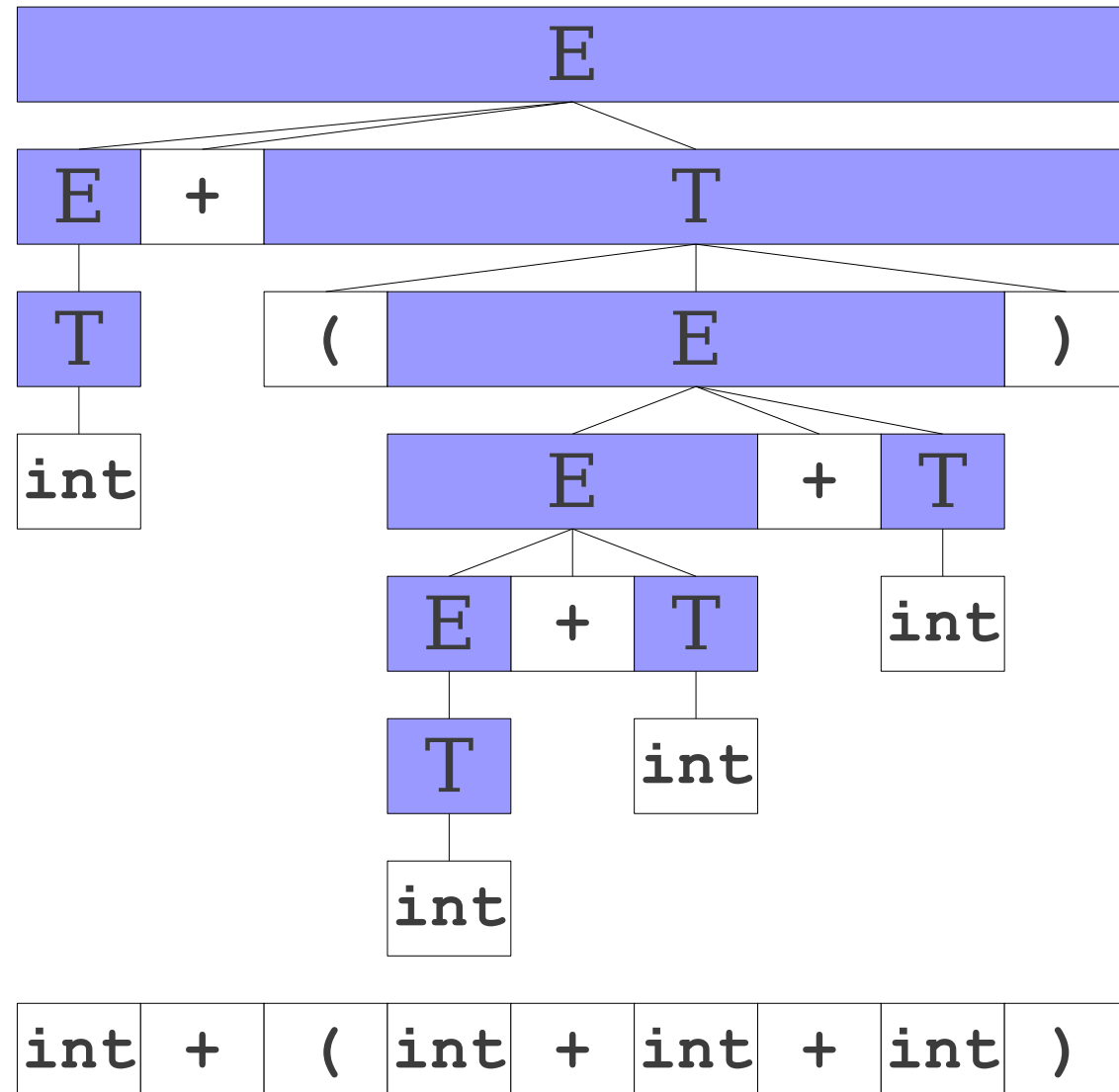
A Third View of a Bottom-Up Parse

`int + (int + int + int)`
⇒ `T + (int + int + int)`
⇒ `E + (int + int + int)`
⇒ `E + (T + int + int)`
⇒ `E + (E + int + int)`
⇒ `E + (E + T + int)`
⇒ `E + (E + int)`
⇒ `E + (E + T)`
⇒ `E + (E)`
⇒ `E + T`
⇒ `E`



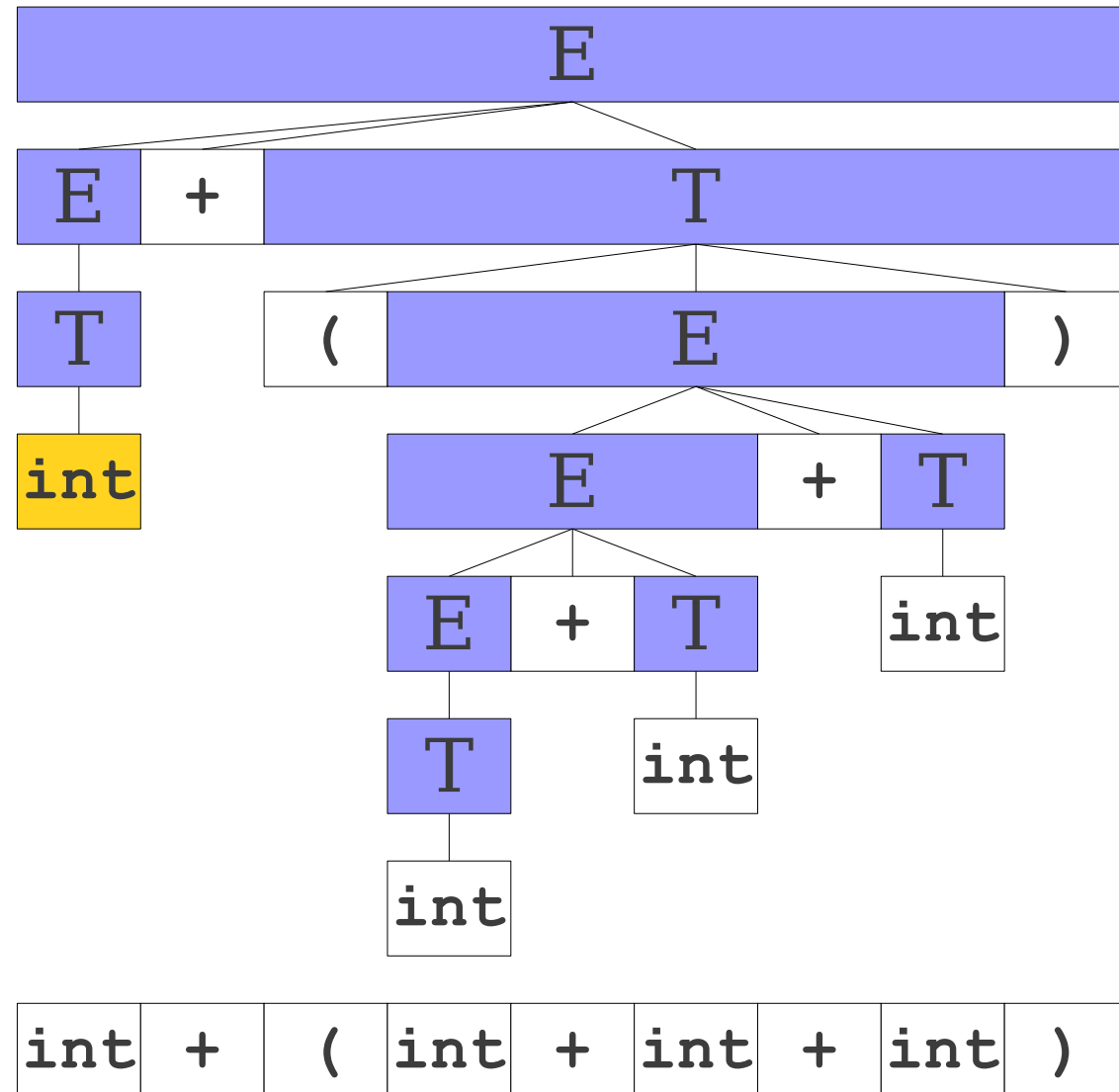
A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ **T** + (int + int + int)
⇒ **E** + (int + int + int)
⇒ **E** + (**T** + int + int)
⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



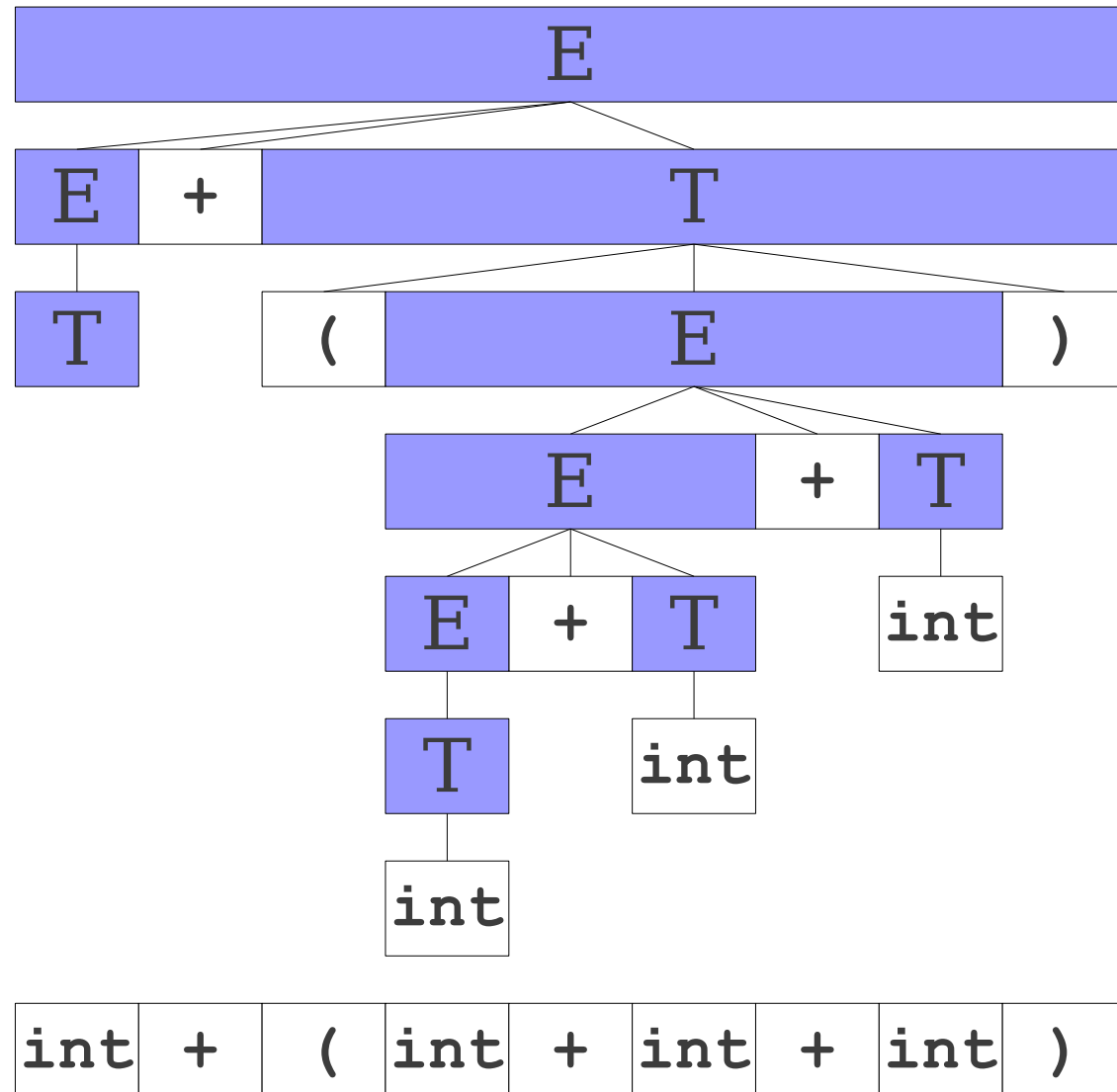
A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ **T** + (int + int + int)
⇒ **E** + (int + int + int)
⇒ **E** + (**T** + int + int)
⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



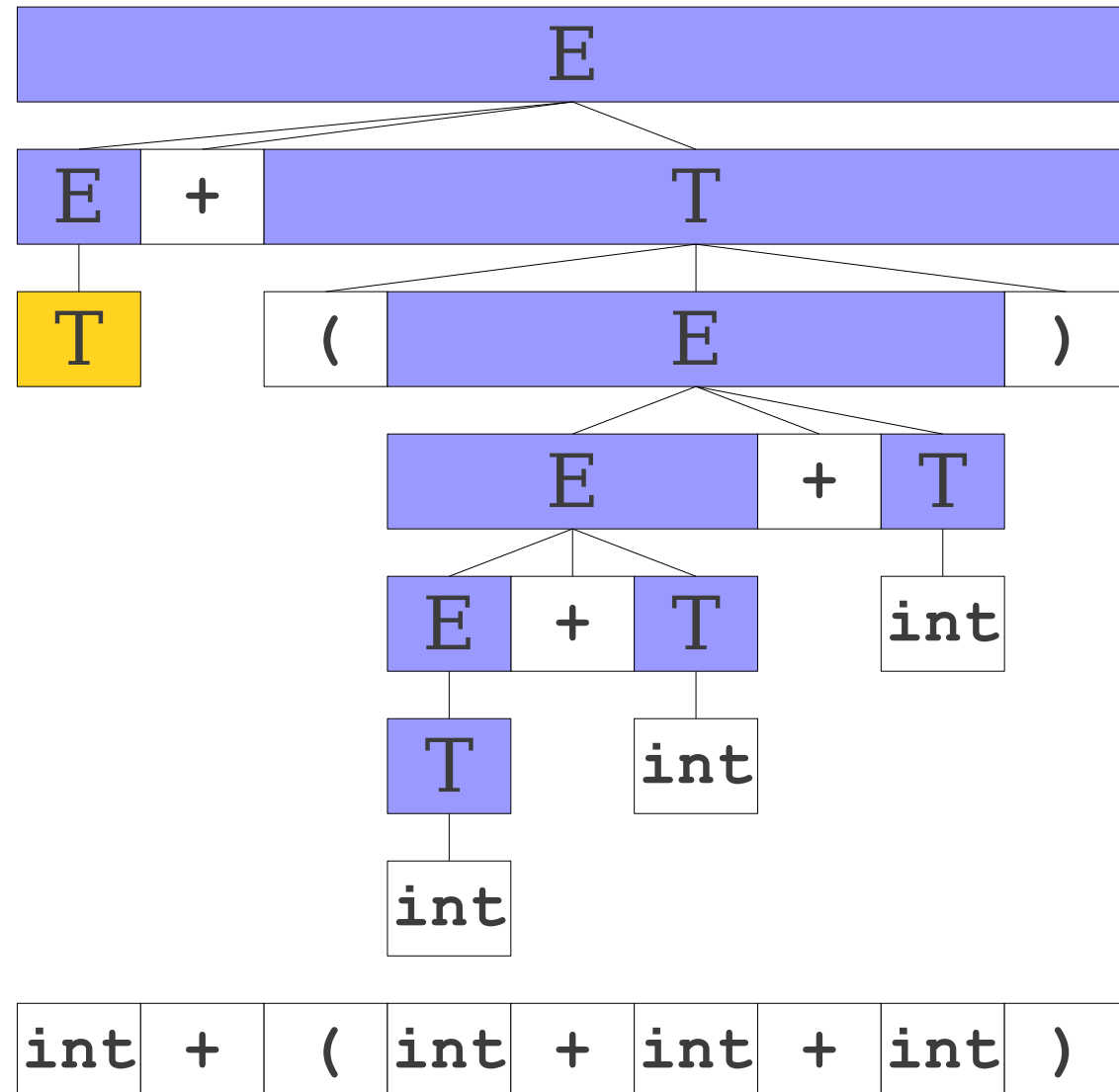
A Third View of a Bottom-Up Parse

$\Rightarrow T + (int + int + int)$
 $\Rightarrow E + (int + int + int)$
 $\Rightarrow E + (T + int + int)$
 $\Rightarrow E + (E + int + int)$
 $\Rightarrow E + (E + T + int)$
 $\Rightarrow E + (E + int)$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



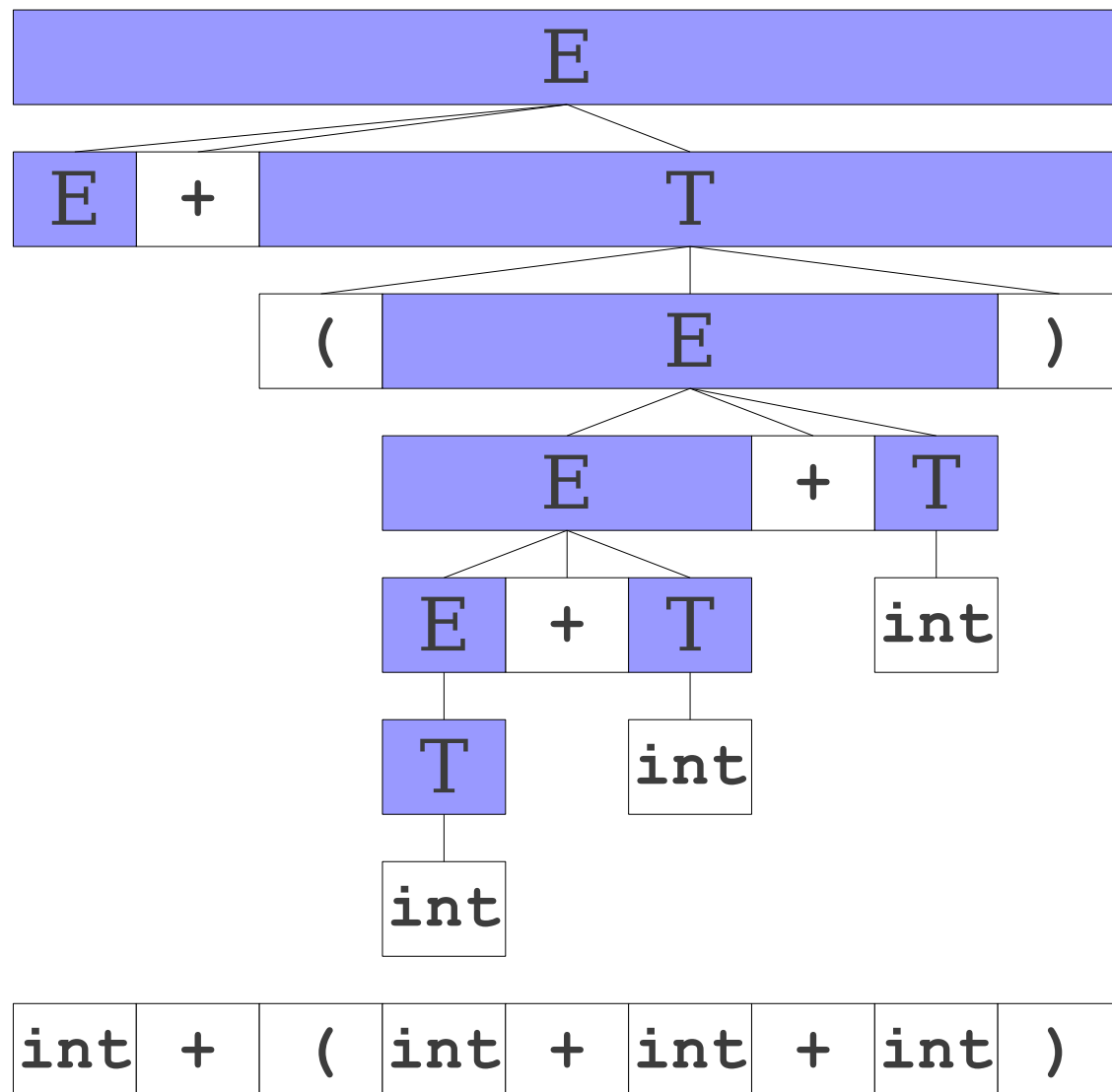
A Third View of a Bottom-Up Parse

$\Rightarrow$ **T** + (int + int + int)
 $\Rightarrow$ **E** + (int + int + int)
 $\Rightarrow$ **E** + (**T** + int + int)
 $\Rightarrow$ **E** + (**E** + int + int)
 $\Rightarrow$ **E** + (**E** + **T** + int)
 $\Rightarrow$ **E** + (**E** + int)
 $\Rightarrow$ **E** + (**E** + **T**)
 $\Rightarrow$ **E** + (**E**)
 $\Rightarrow$ **E** + **T**
 $\Rightarrow$ **E**



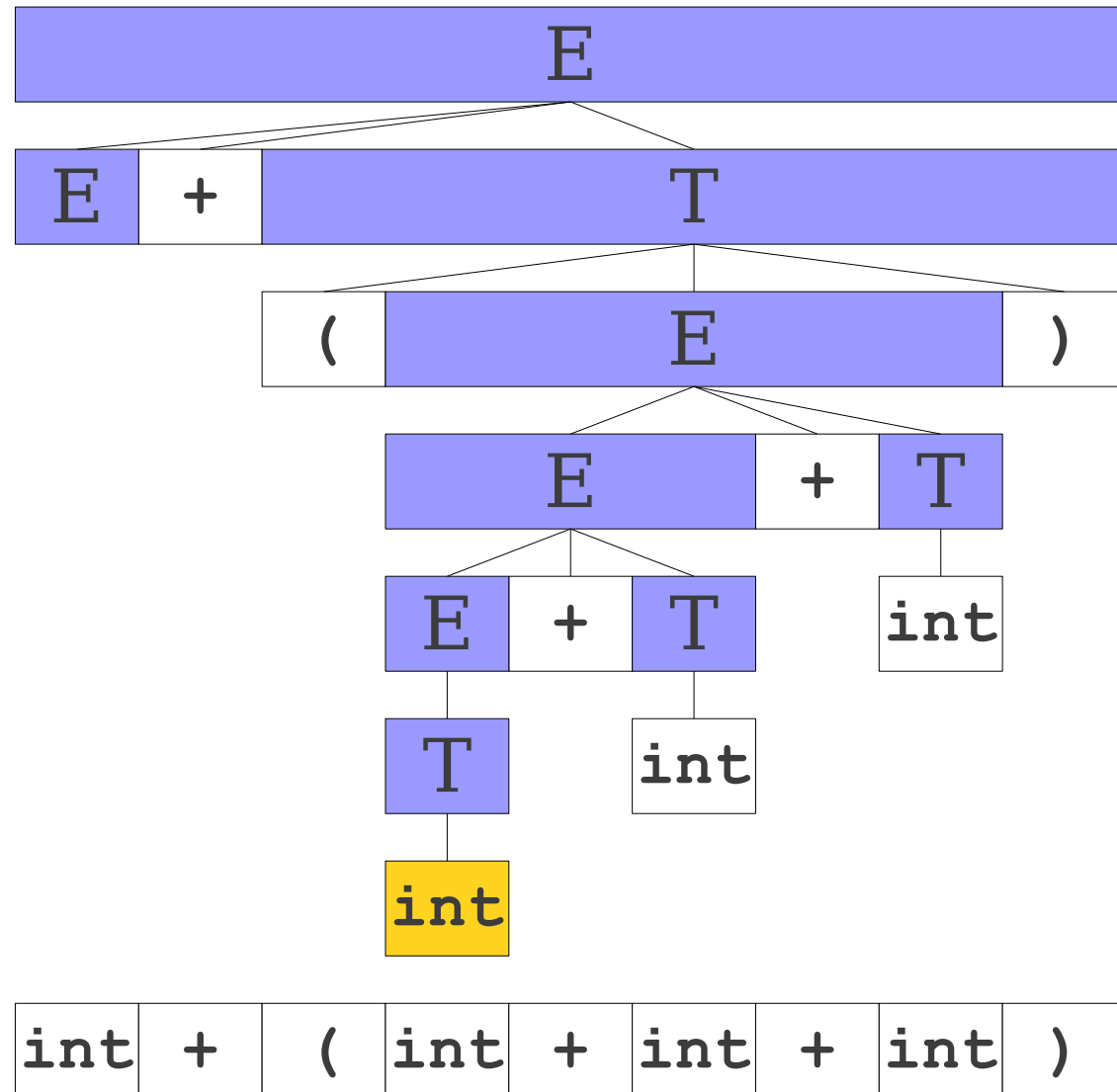
A Third View of a Bottom-Up Parse

$\Rightarrow E + (int + int + int)$
 $\Rightarrow E + (T + int + int)$
 $\Rightarrow E + (E + int + int)$
 $\Rightarrow E + (E + T + int)$
 $\Rightarrow E + (E + int)$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



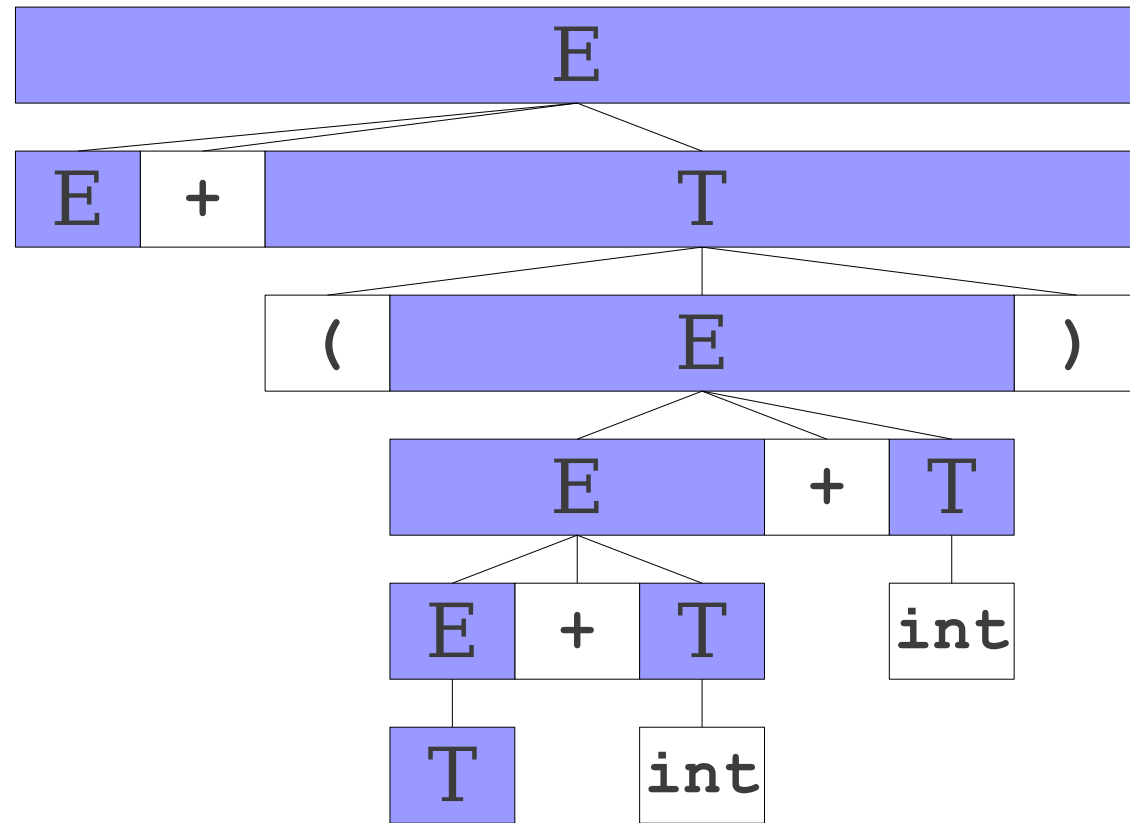
A Third View of a Bottom-Up Parse

$\Rightarrow E + (\text{int} + \text{int} + \text{int})$
 $\Rightarrow E + (T + \text{int} + \text{int})$
 $\Rightarrow E + (E + \text{int} + \text{int})$
 $\Rightarrow E + (E + T + \text{int})$
 $\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



A Third View of a Bottom-Up Parse

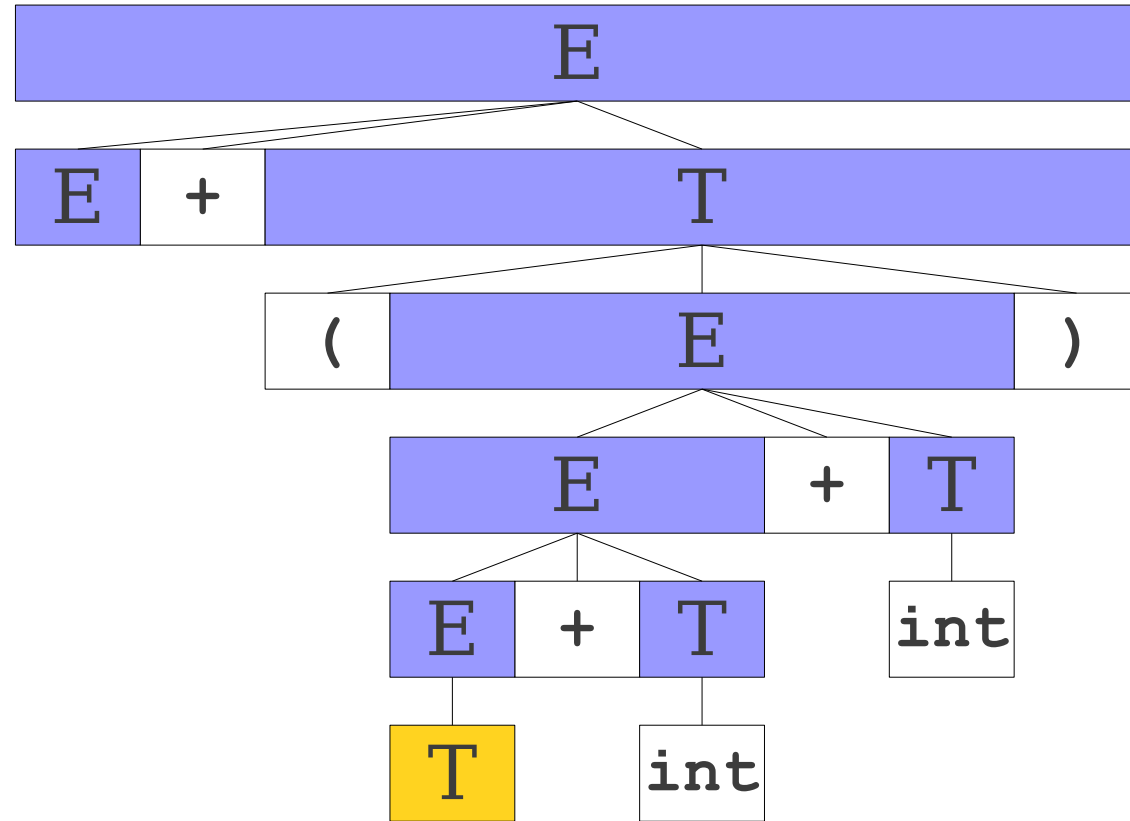
$\Rightarrow E + (T + \text{int} + \text{int})$
 $\Rightarrow E + (E + \text{int} + \text{int})$
 $\Rightarrow E + (E + T + \text{int})$
 $\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

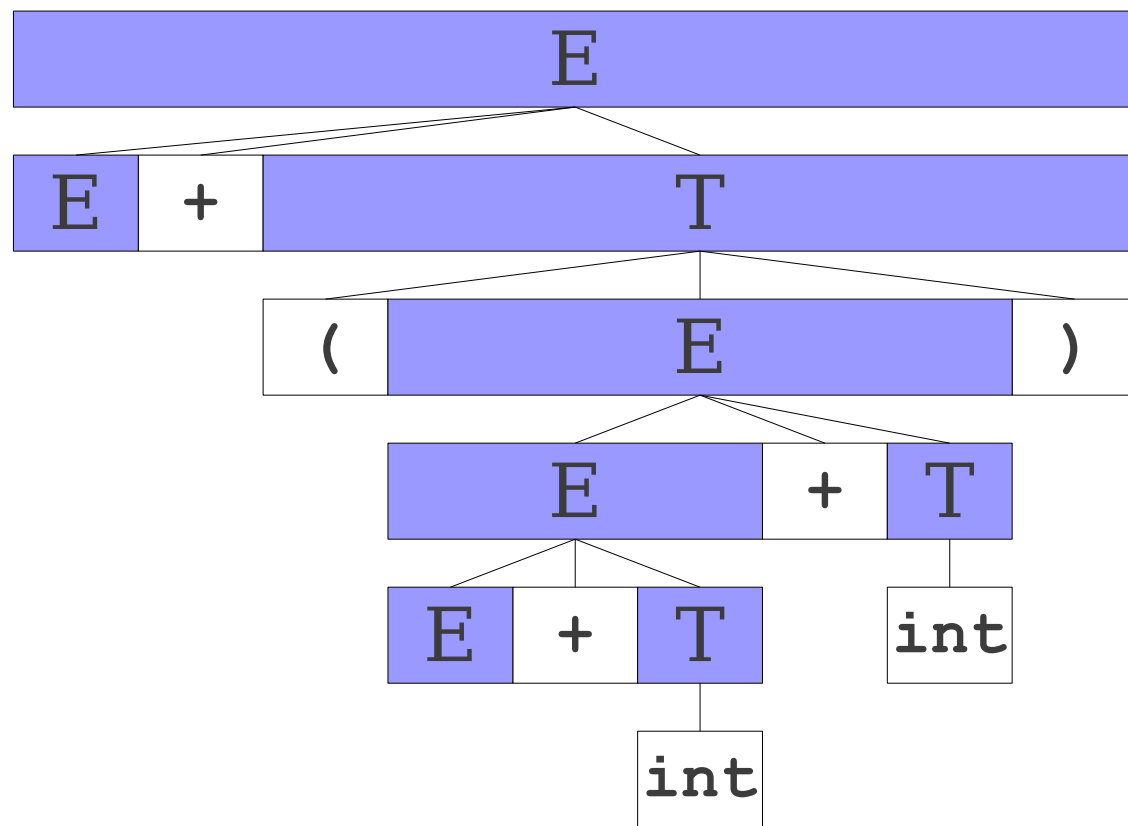
A Third View of a Bottom-Up Parse

$\Rightarrow E + (T + \text{int} + \text{int})$
 $\Rightarrow E + (E + \text{int} + \text{int})$
 $\Rightarrow E + (E + T + \text{int})$
 $\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

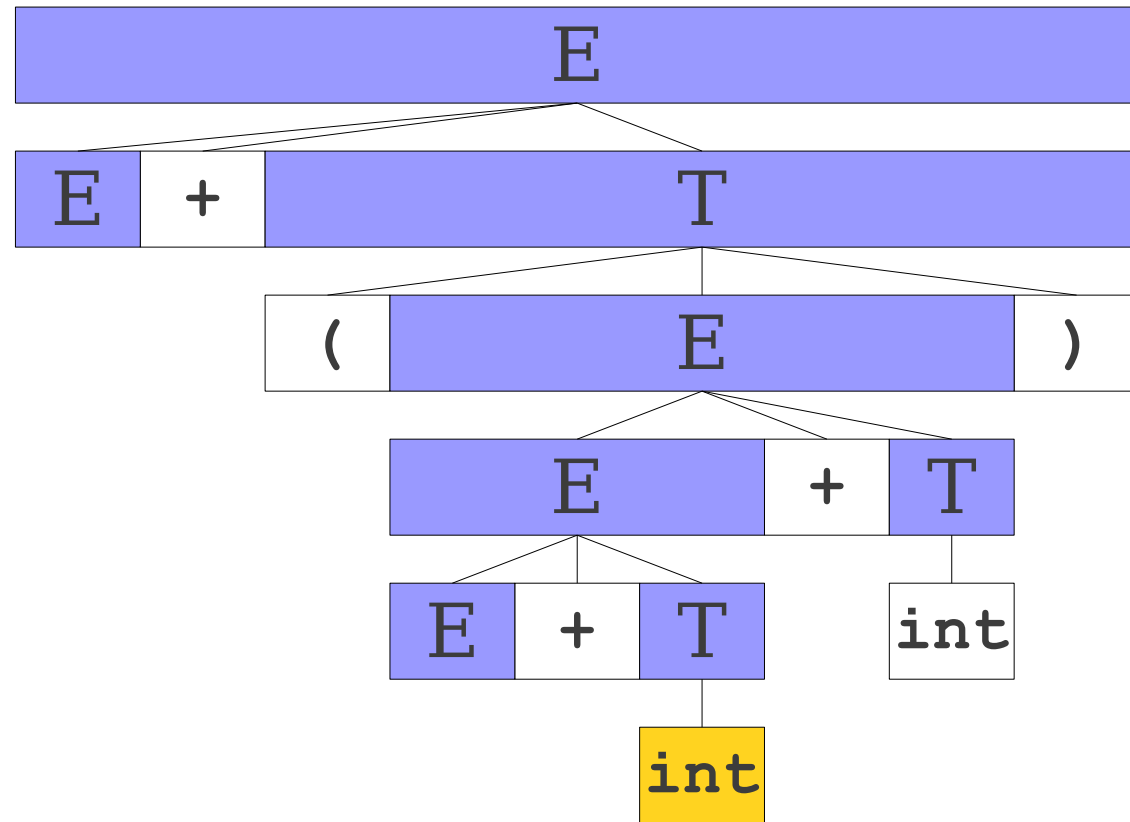
A Third View of a Bottom-Up Parse


$$\begin{aligned} &\Rightarrow E + (E + \text{int} + \text{int}) \\ &\Rightarrow E + (E + T + \text{int}) \\ &\Rightarrow E + (E + \text{int}) \\ &\Rightarrow E + (E + T) \\ &\Rightarrow E + (E) \\ &\Rightarrow E + T \\ &\Rightarrow E \end{aligned}$$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

A Third View of a Bottom-Up Parse

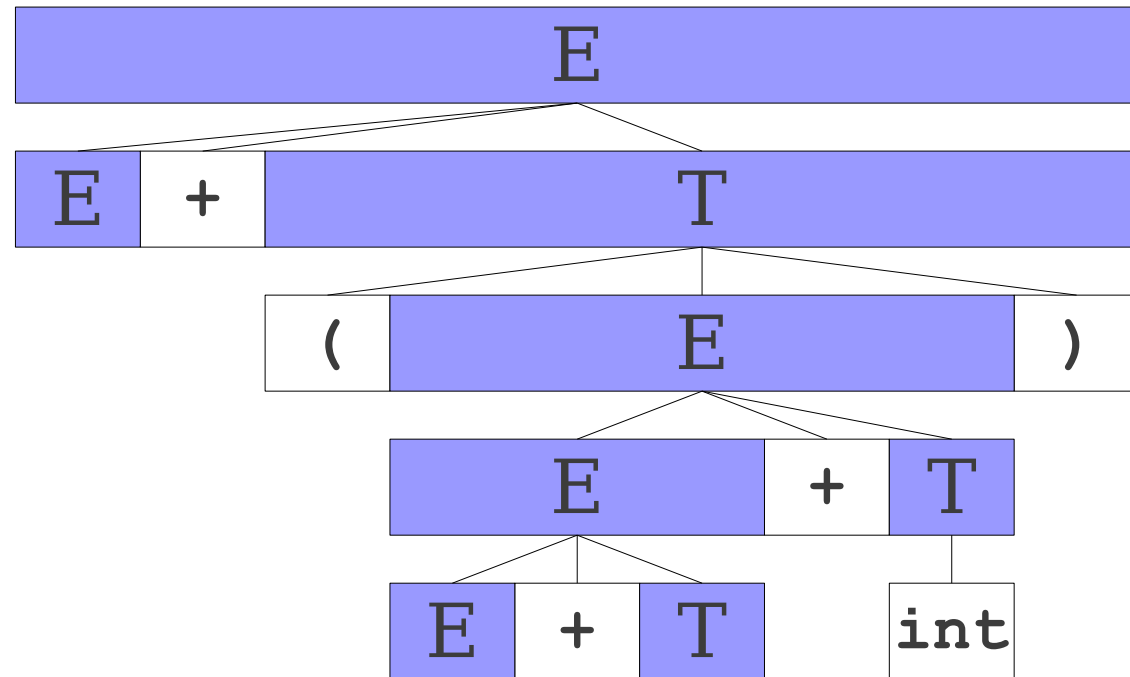
$\Rightarrow E + (E + \text{int} + \text{int})$
 $\Rightarrow E + (E + T + \text{int})$
 $\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

A Third View of a Bottom-Up Parse

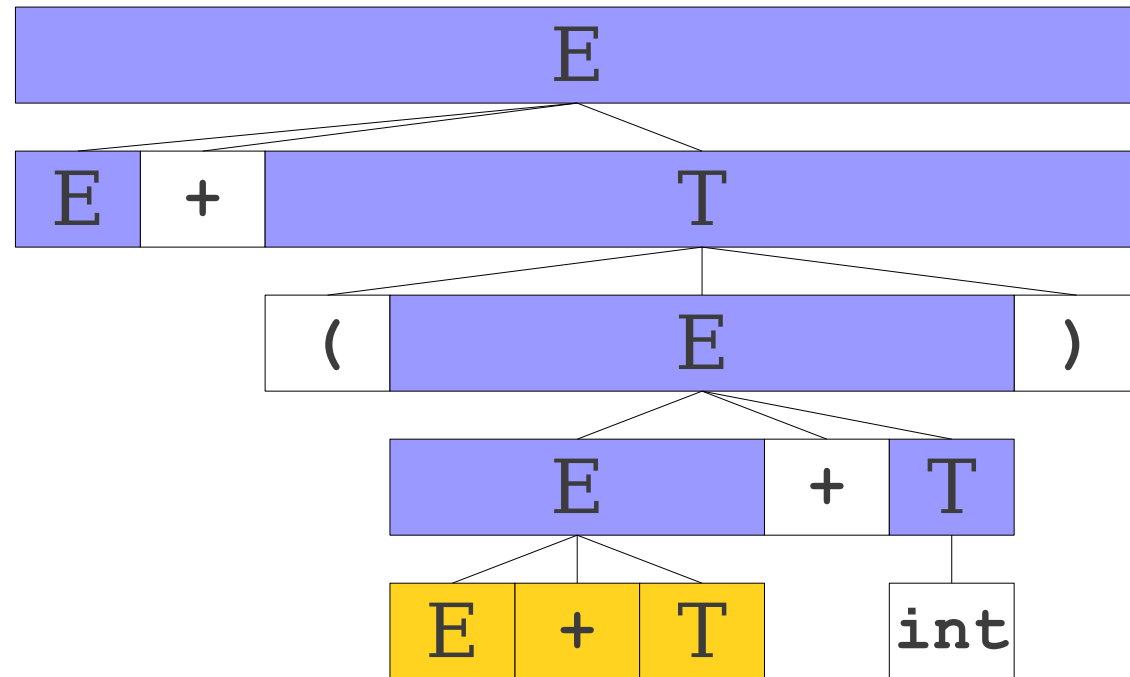
$\Rightarrow E + (E + T + \text{int})$
 $\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

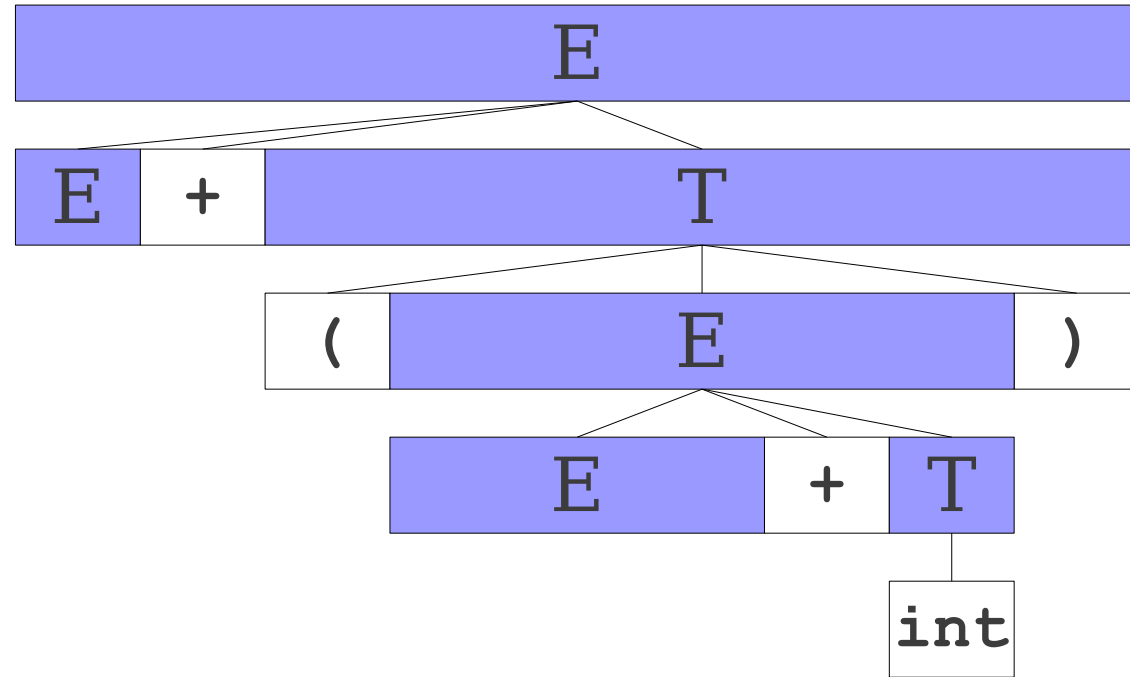
A Third View of a Bottom-Up Parse

$\Rightarrow E + (E + T + \text{int})$
 $\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$



int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

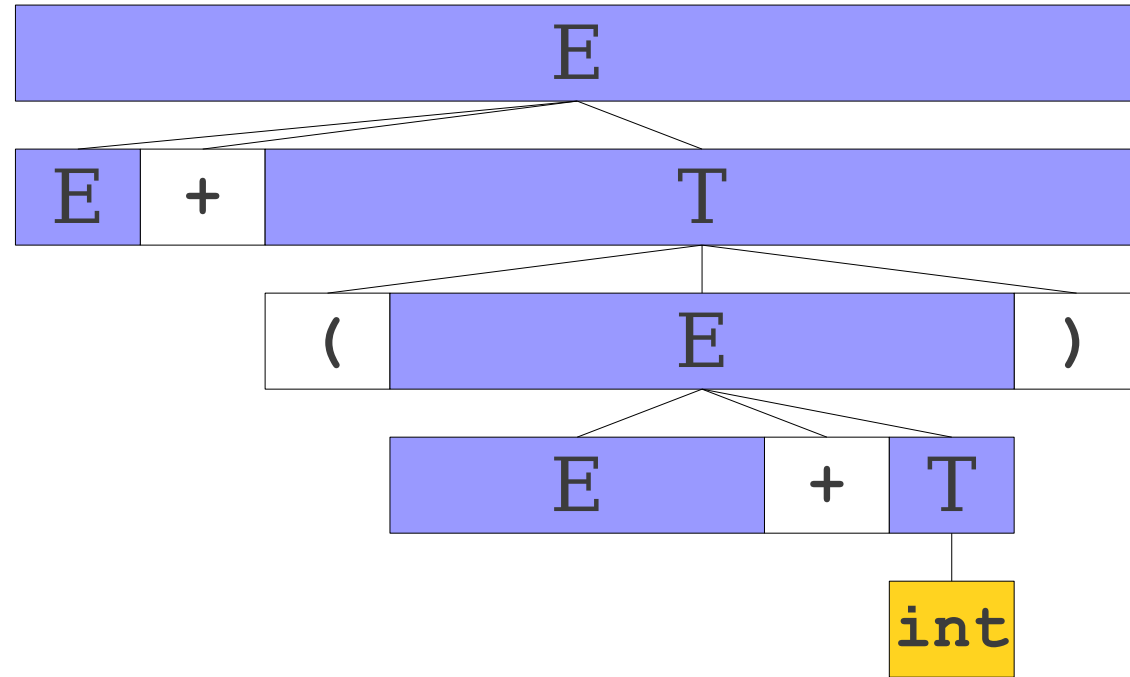
A Third View of a Bottom-Up Parse



$\Rightarrow \mathbf{E} + (\mathbf{E} + \mathbf{int})$
 $\Rightarrow \mathbf{E} + (\mathbf{E} + \mathbf{T})$
 $\Rightarrow \mathbf{E} + (\mathbf{E})$
 $\Rightarrow \mathbf{E} + \mathbf{T}$
 $\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

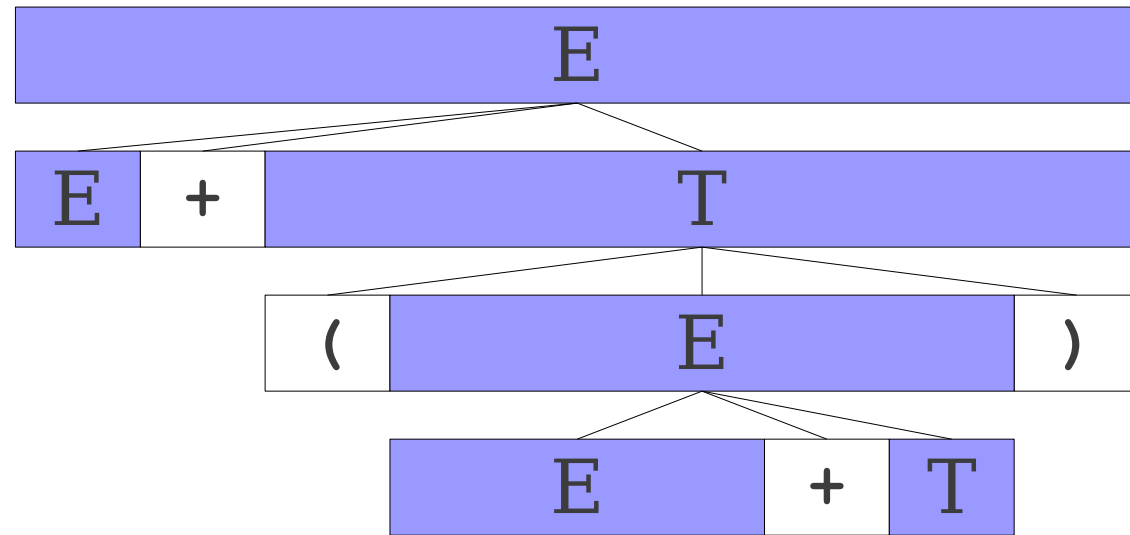
A Third View of a Bottom-Up Parse



$\Rightarrow E + (E + \text{int})$
 $\Rightarrow E + (E + T)$
 $\Rightarrow E + (E)$
 $\Rightarrow E + T$
 $\Rightarrow E$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

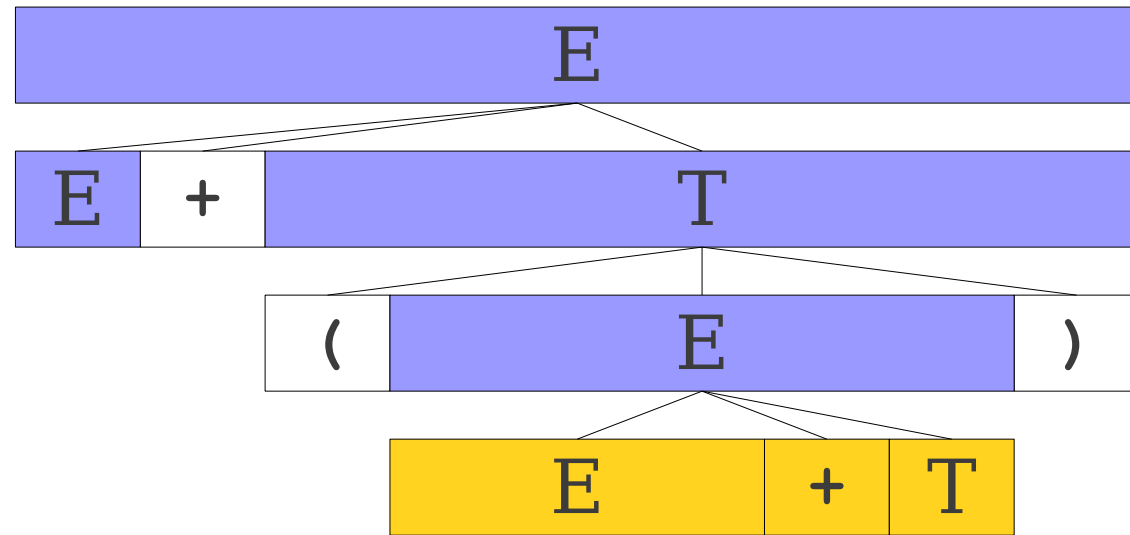
A Third View of a Bottom-Up Parse



$\Rightarrow \mathbf{E} + (\mathbf{E} + \mathbf{T})$
 $\Rightarrow \mathbf{E} + (\mathbf{E})$
 $\Rightarrow \mathbf{E} + \mathbf{T}$
 $\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

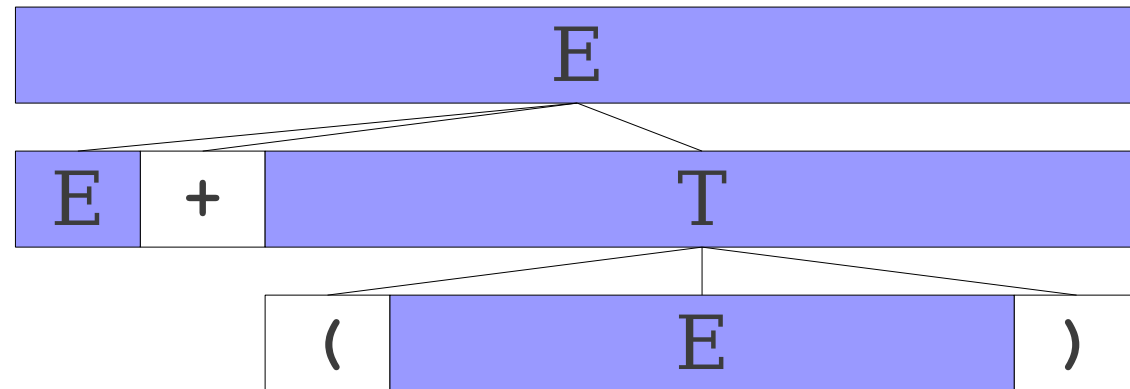
A Third View of a Bottom-Up Parse



$\Rightarrow \mathbf{E} + (\mathbf{E} + \mathbf{T})$
 $\Rightarrow \mathbf{E} + (\mathbf{E})$
 $\Rightarrow \mathbf{E} + \mathbf{T}$
 $\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

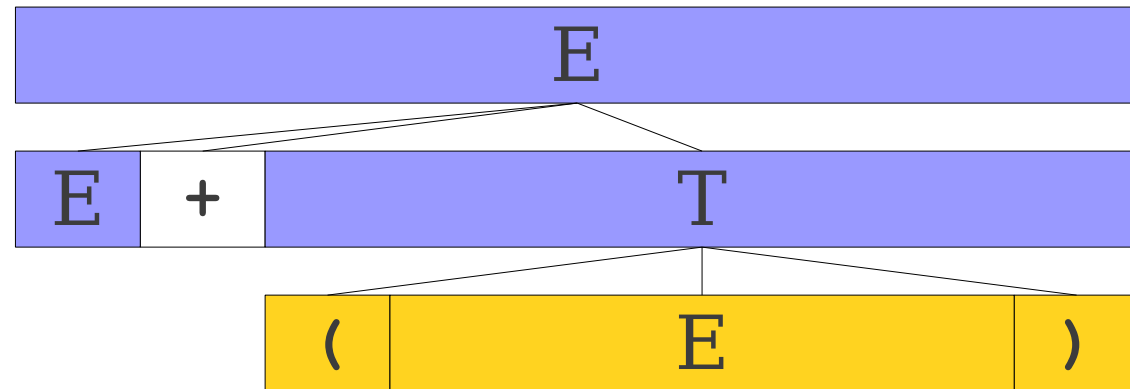
A Third View of a Bottom-Up Parse



$\Rightarrow \mathbf{E} + (\mathbf{E})$
 $\Rightarrow \mathbf{E} + \mathbf{T}$
 $\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

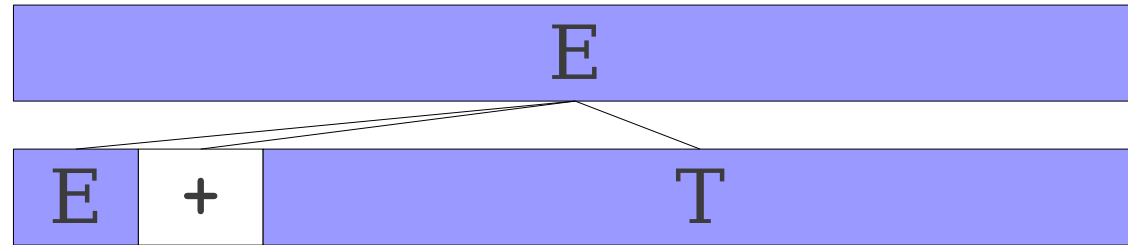
A Third View of a Bottom-Up Parse



$\Rightarrow \mathbf{E} + (\mathbf{E})$
 $\Rightarrow \mathbf{E} + \mathbf{T}$
 $\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

A Third View of a Bottom-Up Parse

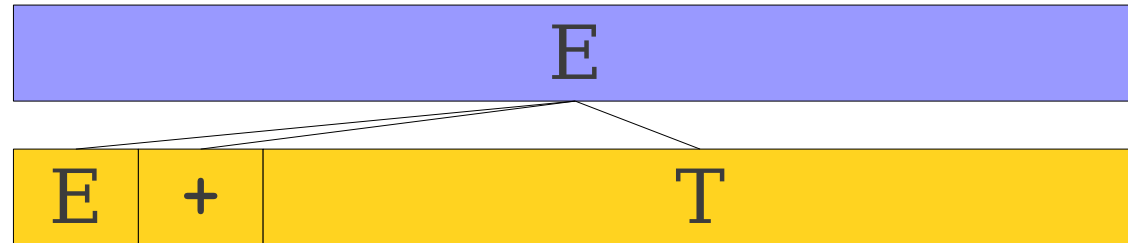


$\Rightarrow \mathbf{E} + \mathbf{T}$

$\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

A Third View of a Bottom-Up Parse



$\Rightarrow \mathbf{E} + \mathbf{T}$

$\Rightarrow \mathbf{E}$

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---

A Third View of a Bottom-Up Parse

E

⇒ E

int	+	(	int	+	int	+	int	)
-----	---	---	-----	---	-----	---	-----	---