

CSE 101 Homework 4
Greedy Algorithms
Due Tuesday, April 30

Directions: For each of the first two problems, two "high level" greedy strategies are given. One of the strategies gives a correct (optimal) solution, and for others, it sometimes gives incorrect (suboptimal) solutions. For each, A (5 points): decide which greedy strategy produces optimal solutions. B (10 points): For the one that is incorrect, give a counter-example showing that it is sometimes incorrect. C (5 points): Give the solution found by the correct strategy on this example. D (10 points): Prove that the correct strategy is indeed correct. E (10 points): Give an efficient version of the correct strategy, and time analysis.

Oxen pairing Consider the following problem: We have n oxen, $Ox_1, \dots, Ox_n$, each with a strength rating S_i . We need to pair the oxen up into teams to pull a plow; if Ox_i and Ox_j are in a team, we must have $S_i + S_j \geq P$, where P is the weight of a plow. Each ox can only be in at most one team. Each team has exactly two oxen. We want to maximize the number of teams.

Candidate Greedy Strategy I: Take the strongest and weakest oxen. If together they meet the strength requirement, make them a team. Recursively find the most teams among the remaining oxen.

Otherwise, delete the weakest ox. Recursively find the most teams among the remaining oxen.

Candidate Greedy Strategy II: Take the weakest two oxen, If together they meet the strength requirement, make them a team. Recursively find the most teams among the remaining oxen.

Otherwise, delete the weakest ox. Recursively find the most teams among the remaining oxen.

Spectrum You want to create a scientific laboratory capable of monitoring any frequency in the electromagnetic spectrum between L and H . You have a list of possible monitoring technologies, T_i , $i = 1, \dots, n$, each with an interval $[l_i, h_i]$ of frequencies that it can be used to monitor. You want to pick as few as possible technologies that together cover the interval $[L, H]$.

Candidate Greedy Strategy I: First, buy the technology that covers the longest sub-interval within (L, H) . (i.e., The longest interval (l, h) , but not including the sub-intervals (l, L) and (H, h) outside the interval we need covering.) At each subsequent step, buy the technology that covers the largest total length that is still uncovered.

Candidate Greedy Strategy II: Look at all the technologies with $l \leq L$. Of these, buy the one $T_i = (l_i, h_i)$ with the largest value of h_i . Repeat the process to cover the remaining interval, (h_i, H) .