CSE 20: Lecture 7
Boolean Algebra

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Outline

1. Introduction
2. Definitions

   Interpretation of Set Operations

   Interpretation of Logic Operations
3. Theorems and Proofs

   Multi-valued Boolean Algebra
4. Transformations

1. Introduction: iClicker

Boolean algebra can be used for:

A. Set operation
B. Logic operation
C. Software verification
D. Hardware designs
E. All of the above.

1. Introduction

Boolean algebra can be used for:

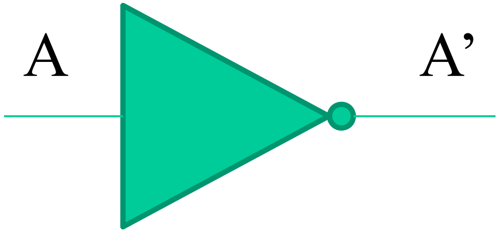
A. Set operation (union, intersect, exclusion)
B. Logic operation (AND, OR, NOT)
C. Software verification
D. Hardware designs (control, data process)

Introduction: Basic Components

We use binary bits to represent true or false.
A=1: A is true
A=0: A is false
We use AND, OR, NOT gates to operate the logic.

NOT gate inverts the value (flip 0 and 1)

$$y = NOT(A) = A'$$



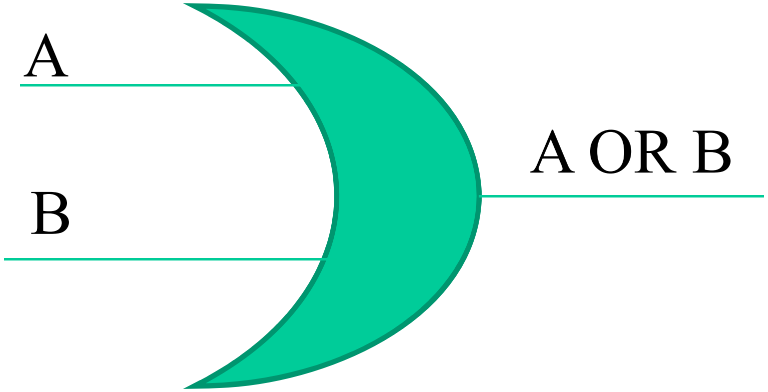
id	**A**	***NOT A***
0	0	1
1	1	0

Introduction: Basic Components

OR gate: Output is true if either input is true

y= A OR B

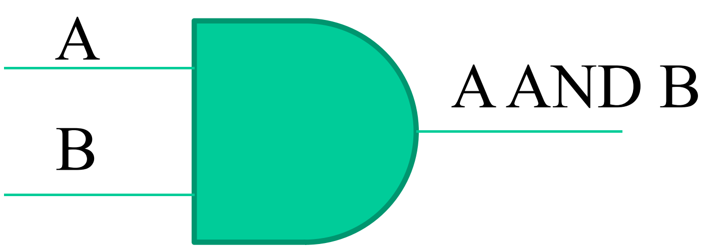
id	**A B**	***A OR B***
0	0 0	0
1	0 1	1
2	1 0	1
3	1 1	1



Introduction: Basic Components

AND gate: Output is true only if all inputs are true

y= A AND B

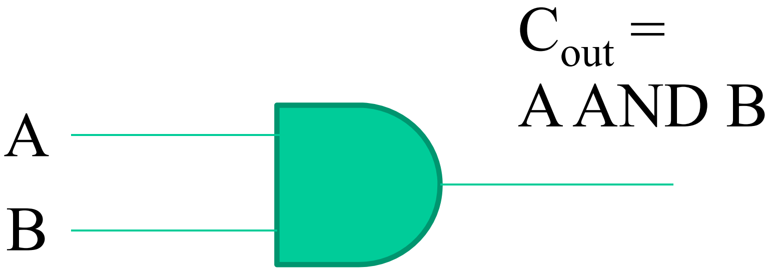


Id	**A**	**B**	***A AND B***
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1

Introduction: Half Adder

A Half Adder:
Carry = A AND B
Sum = (A AND B') OR (A' AND B)

id	A, B	C_{out}, S_{um}
0	00	0 0
1	01	0 1
2	10	0 1
3	11	1 0

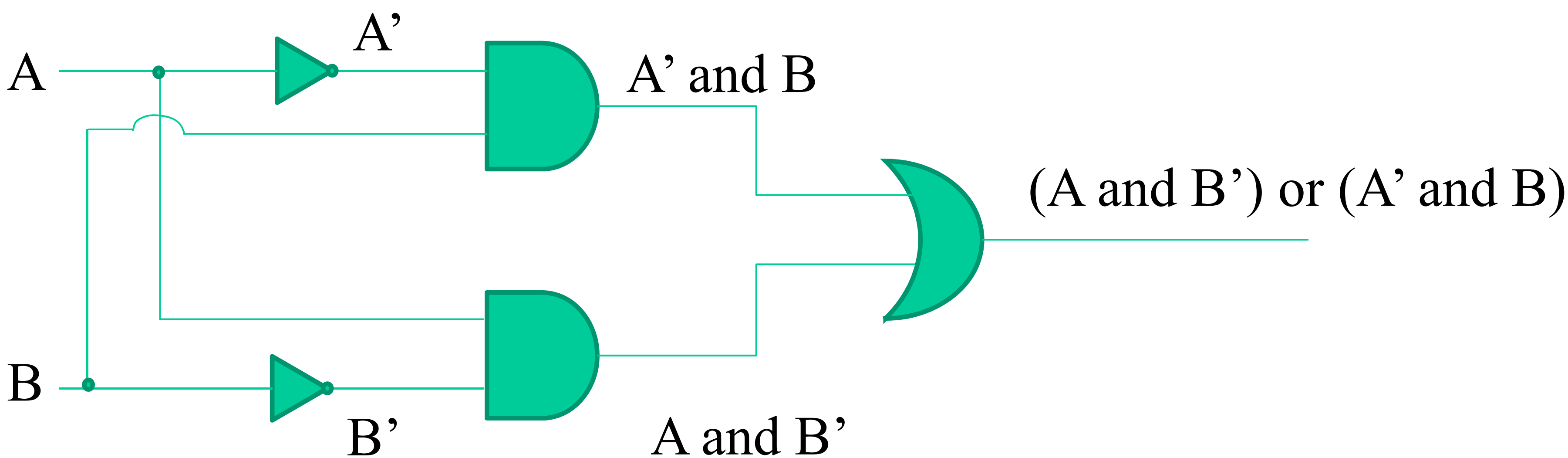


Introduction: Half Adder

A Half Adder:

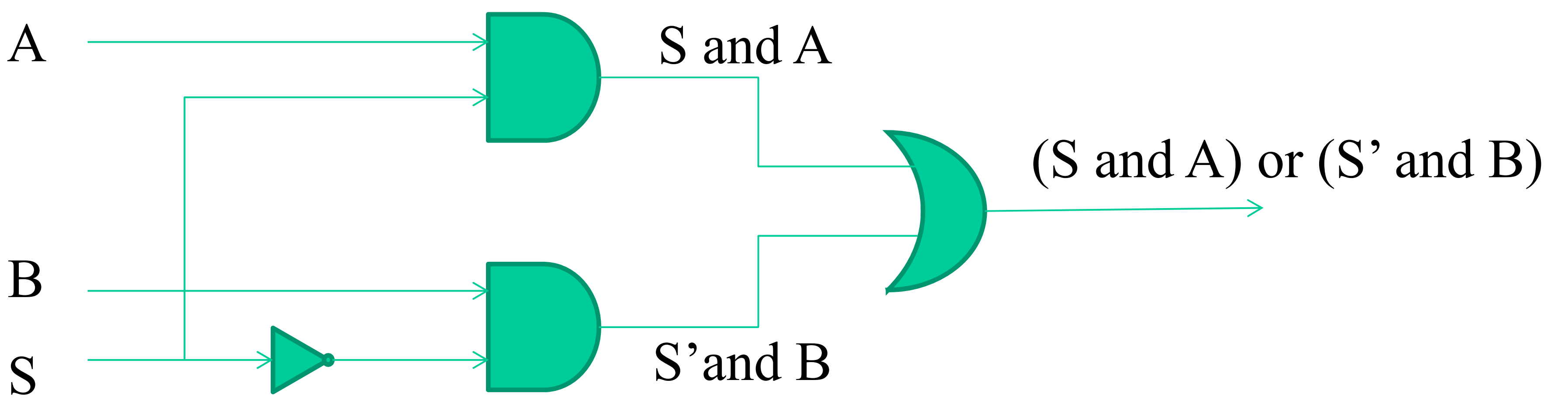
C_{out} = A AND B
S_{um} = (A AND B') OR (A' AND B)

S_{um}:



Introduction: Multiplexer

A multiplexer:
If S then Z=A else Z=B



2. Definition

Boolean Algebra: A set of elements B with two operations. + (OR, U, ∨)

* (AND, ∩, ∧),

satisfying the following 4 laws for every a, b, c in B.

P1. Commutative Laws:

a+b = b+a; a*b = b*a,

P2. Distributive Laws:

a+(b*c) = (a+b)*(a+c); a*(b+c)= (a*b)+(a*c),

P3. Identity Elements: Set B has two distinct elements denoted as 0 and 1, such that a+0 = a; a*1 = a,

P4. Complement Laws:

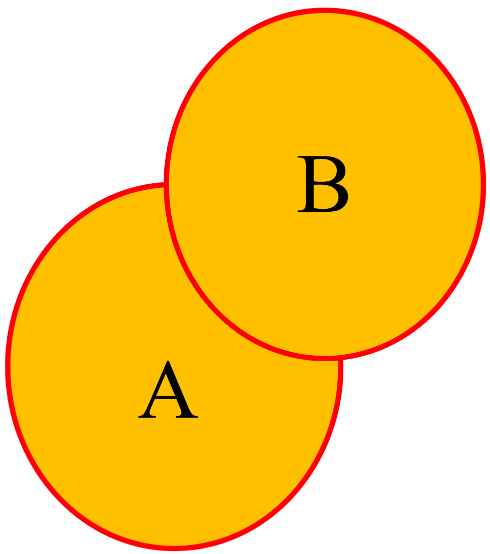
a+a’ = 1; a*a’ = 0.

Interpretation of Set Operations

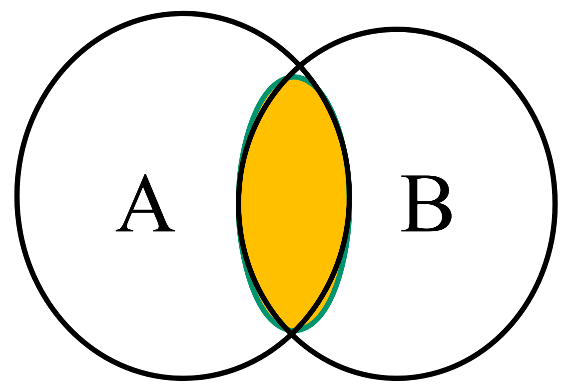
- Set: Collection of Objects
- Example:
- $A = \{1, 3, 5, 7, 9\}$
- $N = \{x \mid x \text{ is a positive integer}\}$, e.g. $\{1, 2, 3, \ldots\}$
- $Z = \{x \mid x \text{ is an integer}\}$, e.g. $\{-1, 0, 4\}$
- $Q = \{x \mid x \text{ is a rational number}\}$, e.g. $\{-0.75, \frac{2}{3}, 100\}$
- $R = \{x \mid x \text{ is a real number}\}$, e.g. $\{\pi, 12, -\frac{1}{3}\}$
- $C = \{x \mid x \text{ is a complex number}\}$, e.g. $\{2 + 7i\}$
- $\Phi = \{\}$ or empty set

P1. Commutative Laws in Venn Diagram

A U B= B U A

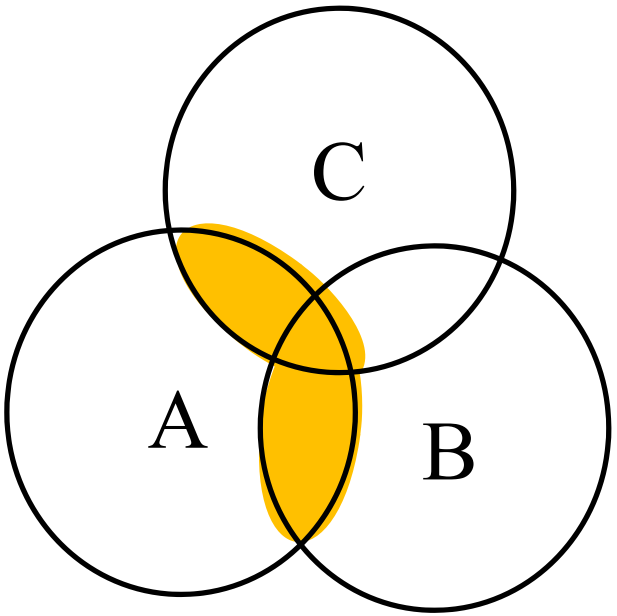


A∩B= B∩A



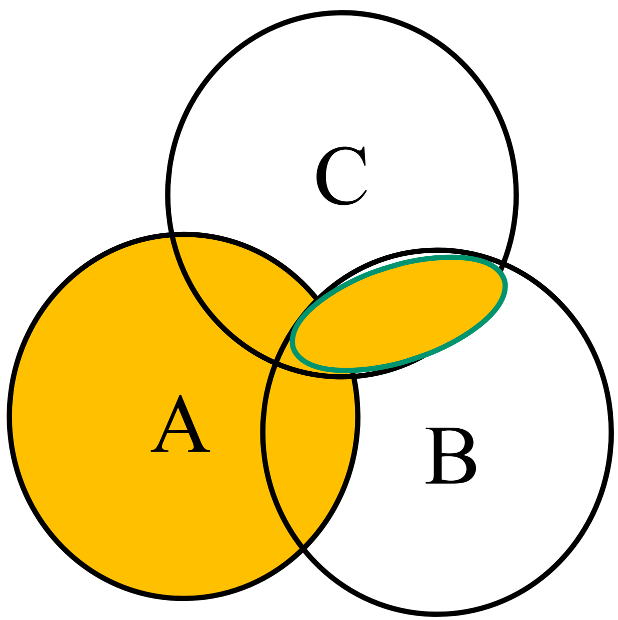
P2. Distributive Laws

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



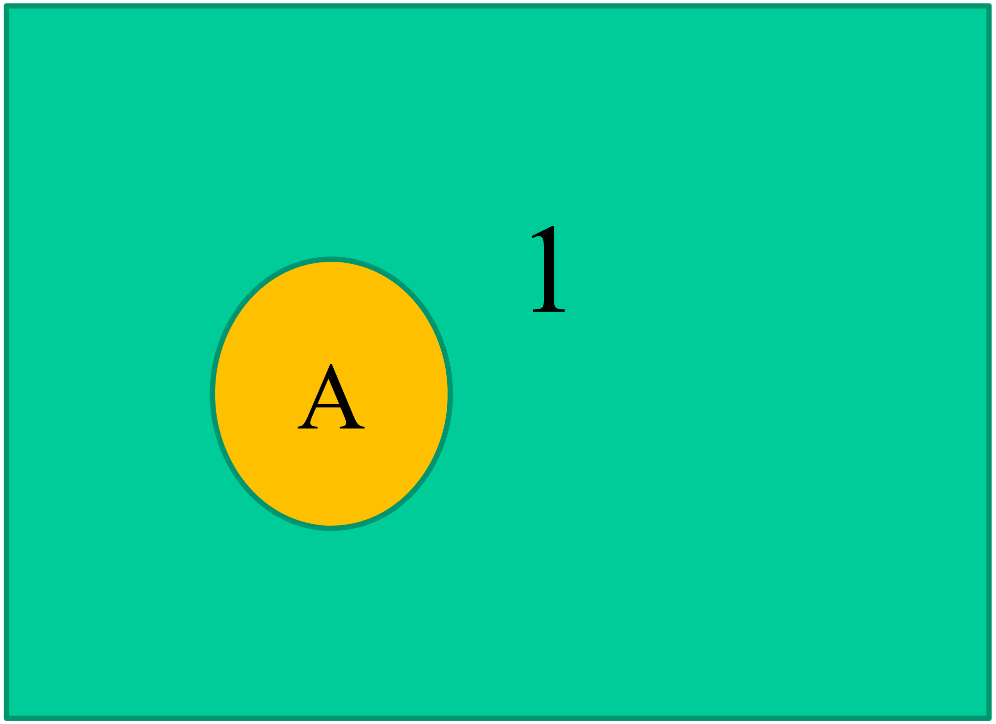
P2. Distributive Laws

- A U (B∩C) = (A U B)∩(A U C)



P3. Identity Elements

- 0 = {}
- 1 = Universe of the set
- A U 0 = A
- A∩1 = A



P4: Complement

- $A \cup A' = 1$
- $A \cap A' = 0$