# CSE20

## Lecture 2: Number Systems: Binary Numbers and Gray Code

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# Number Systems

1. Introduction
2. Binary Numbers
3. Gray code
4. Negative Numbers
5. Residual Numbers

# 2. Binary Numbers: iClicker

What is the extent of a binary number system

- A. Coverage of integer and floating point numbers
- B. Mechanism of addition and subtraction operations
- C. Operations of logic functions
- D. All of the above
- E. None of the above.

# 2. Binary Numbers

1. Definition (radix 2)
2. Enumerations (value -> index)
3. Addition (logic -> hardware)

# 2.1 Definition of Binary Numbers

- Format: An n digit binary number

$(b_{n-1}, \ldots, b_1, b_0)_2$ where $b_i$ in {0,1} for $0 <= i < n$

- Value: $b_{n-1}2^{n-1} + \ldots + b_1 2^1 + b_0 2^0$
- Non-redundancy: The system is non-redundant, i.e. different binary numbers represent different values.

# 2.2 Enumeration of Binary Numbers

1 digit

| id | $b_0$ |
|---|---|
| 0 | 0 |
| 1 | 1 |

2 digits

| id | $b_1b_0$ |
|---|---|
| 0 | 00 |
| 1 | 01 |
| 2 | 10 |
| 3 | 11 |

3 digits

| id | $b_2b_1b_0$ |
|---|---|
| 0 | 000 |
| 1 | 001 |
| 2 | 010 |
| 3 | 011 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |

4 digits?

| id | $b_3b_2b_1b_0$ |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| . | . |
| . | . |
| . | . |
| 14 | 1110 |
| 15 | 1111 |

An n digit binary code covers numbers from 0 to $2^n$-1.

# 2.2 Enumeration of binary numbers iCliker

When we enumerate binary numbers $(b_3b_2b_1b_0)_2$ from 0 to 15, the sequence of $b_3$ should be

- A. 0101010101010101
- B. 0011001100110011
- C. 0000111100001111
- D. 0000000011111111

# 2.3 Addition of Binary Numbers

Given two binary numbers A & B, we derive binary number S so that the value of S is equal to the sum of the values of A & B, i.e.

$$(a_{n-1}\ldots,a_1a_0)_2+(b_{n-1}\ldots b_1b_0)_2=(s_{n-1}\ldots s_1s_0)_2$$

Caution: Overflow, i.e. the sum is beyond the range of the representation.

# 2.3 Addition: iClicker

Given two binary numbers

$A=(a_{n-1}\ldots,a_1a_0)_2$ and $B=(b_{n-1}\ldots b_1b_0)_2$

what is the largest possible value of A+B?

A. $2^{n+1}$

B. $2^{n+1}-1$

C. $2^{n+1}-2$

D. None of the above

# 2.3 Addition of Binary Numbers

Equality of addition

$(a_{n-1}\ldots,a_1a_0)_2+(b_{n-1}\ldots b_1b_0)_2=(s_{n-1}\ldots s_1s_0)_2$

That is to say

$a_{n-1}2^{n-1}+\ldots+a_12^1+a_02^0+b_{n-1}2^{n-1}+\ldots+b_12^1+b_02^0$

$=(a_{n-1}+b_{n-1})2^{n-1}+\ldots+(a_1+b_1)2^1+(a_0+b_0)2^0$

$=s_{n-1}2^{n-1}+\ldots+s_12^1+s_02^0$

# 2.3 Addition of Binary Numbers

| $b_2$ | $b_1$ | $b_0$ | Value |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 4 |
| 1 | 0 | 1 | 5 |
| 1 | 1 | 0 | 6 |
| 1 | 1 | 1 | 7 |

Examples:

| | *8* | *4* | *2* | *1* |
|---|---|---|---|---|
| | 0 | 0 | 1 | 1 |
| + | 0 | 1 | 0 | 1 |

| | *8* | *4* | *2* | *1* |
|---|---|---|---|---|
| | 0 | 0 | 1 | 1 |
| + | 0 | 1 | 1 | 0 |

# 2.3 Addition of Binary Numbers

| ***Bit$_{i+1}$*** | ***Bit$_i$*** | ***Bit$_{i-1}$*** |
|---|---|---|
| Carry$_{i+1}$ | Carry$_i$ | |
| | $a_i$ | $a_{i-1}$ |
| | $b_i$ | $b_{i-1}$ |
| | Sum$_i$ | Sum$_{i-1}$ |

Formula for Bit i:

$$Carry_i + a_i + b_i = 2 \times Carry_{i+1} + Sum_i$$

# 2.3 Adding 2 bits in a digit

| a | b | Carry | Sum |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Formula:

a+b=

2xCarry + Sum

# 2.3 Adding 3 bits in a digit

| id | a | b | c | Carry | Sum |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 | 0 |
| 6 | 1 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 1 | 1 | 1 |

**Formula:**

**a+b+c=**

**2xCarry + Sum**

# 3. Gray Code

1. Introduction
2. Example
3. Construction
4. Comments

# 3.1 Gray Code: Introduction

Gray: Frank Gray patented the code in 1947

A variation of binary code

The code will be used for logic operation (CSE20, CSE140)

Feature: only one bit changes for two consecutive numbers

# 3.2 Gray Code: Example

2 digits

| id | $b_1b_0$ | $g_1g_0$ |
| --- | --- | --- |
| 0 | 00 | 00 |
| 1 | 01 | 01 |
| 2 | 10 | 11 |
| 3 | 11 | 10 |

3 digits

| id | $b_2b_1b_0$ | $g_2g_1g_0$ |
| --- | --- | --- |
| 0 | 000 | 000 |
| 1 | 001 | 001 |
| 2 | 010 | 011 |
| 3 | 011 | 010 |
| 4 | 100 | 110 |
| 5 | 101 | 111 |
| 6 | 110 | 101 |
| 7 | 111 | 100 |

Note the difference of the first and last rows.

# 3.2 Gray Code

3 digits

| id | $b_2b_1b_0$ | $g_2g_1g_0$ |
|---|---|---|
| 0 | 000 | 000 |
| 1 | 001 | 001 |
| 2 | 010 | 011 |
| 3 | 011 | 010 |
| 4 | 100 | 110 |
| 5 | 101 | 111 |
| 6 | 110 | 101 |
| 7 | 111 | 100 |

| id | $b_3b_2b_1b_0$ | $g_3g_2g_1g_0$ |
|---|---|---|
| 0 | 0000 | 0000 |
| 1 | 0001 | 0001 |
| 2 | 0010 | 0011 |
| 3 | 0011 | 0010 |
| 4 | 0100 | 0110 |
| 5 | 0101 | 0111 |
| 6 | 0110 | 0101 |
| 7 | 0111 | 0100 |
| 8 | 1000 | ? |
| 9 | 1001 | |
| 10 | 1010 | |
| 11 | 1011 | |
| 12 | 1100 | |
| 13 | 1101 | |
| 14 | 1110 | |
| 15 | 1111 | |

# 3.2 Gray Code: iClicker

A 4-digit Gray code ($g_3g_2g_1g_0$) at id=8 is written as (ref: previous page)

A. (0101)

B. (0110)

C. (1100)

D. None of the above

# 3.3 Gray Code: Construction

Construction of n-digit Gray code from n-1 digit Gray code

•Copy the n-1 digit Gray code for the top $2^{n-1}$ rows. Fill 0 at digit $g_{n-1}$ in the top rows.

•Reflect and append the n-1 digit code for the bottom $2^{n-1}$ rows. Fill 1 at digit $g_{n-1}$ in the bottom rows.

# 3.4 Gray Code: Comments

- There are various codes that satisfy the Gray code feature.

- Gray code saves communication power when the signals are continuous in nature, e.g. addresses, analog signals

- Gray code facilitates code checking when the signals are supposed to be continuous in value.
- For arithmetic operations, we need to convert the values.