CSE 20 – Discrete Math
Lecture 10

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Boolean Algebra: Theorems and Proofs

Theorem: (Associative Laws) For elements a, b, c in B, we have $a+(b+c)=(a+b)+c$; $a*(b*c)=(a*b)*c$.

Proof: Denote $x = a+(b+c)$, $y = (a+b)+c$.

We want to show that (1) $ax = ay$, (2) $a'x = a'y$.

Then, we have $x= 1*x=(a+a')x= ax+a'x = ay+a'y= (a+a')y= 1*y=y$

Shannon's Expansion: (divide and conquer) We use a variable *a* to divide the term *x* into two parts *ax* and *a'x*.

Proof of (1) $ax = ay$

$ax = a(a+(b+c)) = a\,a + a(b+c)$ (Distributive)

$= a + a(b+c)$ (Idempotence)

$= a$ (Absorption)

$ay = a((a+b)+c)) = a(a+b) + a\,c$ (Distributive)

$= (aa + ab) + ac$ (Distributive)

$= (a + ab) + ac$ (Idempotence)

$= a + ac$ (Absorption)

$= a$ (Absorption)

Therefore: $ax = a = ay$

Proof of (2) $a'x = a'y$

$a'x = a'(a+(b+c)) = a'a + a'(b+c)$ (Distributive)

$= 0 + a'(b+c)$ (Complementary)

$= a'(b+c)$ (Identity)

$a'y = a'((a+b)+c) = a'(a+b) + a'c$ (Distributive)

$= a'b + a'c$ (Theorem 8)

$= a'(b+c)$ (Distributive)

Therefore: $a'x = a'(b+c) = a'y$

Boolean Transformation

Minimize the expression: $(abc+ab')'(a'b+c')$

$(abc+ab')'(a'b+c')$

$=(a(bc+b'))'(a'b+c')$ (distributive)

$=(a(b'+c))'(a'b+c')$ (absorption)

$=(a'+bc')(a'b+c')$ (De Morgan's)

$=a'b+a'c'+a'bc'+bc'$ (distributive)

$=a'b+a'c'+bc'$ (absorption)

Boolean Transformation (switching function)

- ab'+b'c'+a'c'= ab'+a'c'

Proof: ab'+b'c'+a'c'

= ab'+ab'c'+a'b'c'+a'c'

=ab'+a'c'

y=ab'+a'c'

We can use truth table when B={0,1}, which is the case of digital logic designs.

id	a	b	c	ab'	b'c'	a'c'	f
0	0	0	0	0	1	1	1
1	0	0	1	0	0	0	0
2	0	1	0	0	0	1	1
3	0	1	1	0	0	0	0
4	1	0	0	1	1	0	1
5	1	0	1	1	0	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	0	0	0

Boolean Transformation (switching function)

- ab'+b'c'+a'c'= ab'+ab'c'+a'b'c'+a'c'= ab'+a'c'

id	a	b	c	ab'	b'c'	a'c'	f
0	0	0	0	0	1	1	**1**
1	0	0	1	0	0	0	**0**
2	0	1	0	0	0	1	**1**
3	0	1	1	0	0	0	**0**
4	1	0	0	1	1	0	**1**
5	1	0	1	1	0	0	**1**
6	1	1	0	0	0	0	**0**
7	1	1	1	0	0	0	**0**

b,c	0,0	0,1	1,1	1,0
a=0	1	0	0	1
a=1	1	1	0	0

K Map (CSE140)

Boolean Transformation: iClicker

a’c+ab’+b’c = ?

A. ab’+a’c’
B. a’c+ab’
C. ab’+b’c
D. a’c+b’c
E. None of the above

Boolean Transformation (switching function)

- (a+b)(a+c’)(b’+c’)=(a+b)(b’+c’)

id	a	b	c	a+b	a+c’	b’+c’	**f**
0	0	0	0	0	1	1	**0**
1	0	0	1	0	0	1	**0**
2	0	1	0	1	1	1	**1**
3	0	1	1	1	0	0	**0**
4	1	0	0	1	1	1	**1**
5	1	0	1	1	1	1	**1**
6	1	1	0	1	1	1	**1**
7	1	1	1	1	1	0	**0**

b,c	0,0	0,1	1,1	1,0
a=0	0	0	0	1
a=1	1	1	0	1

K Map (CSE140)

Summary of Boolean Algebra

- Definition: a set B, two operations and four postulates.
- Applicability: set operations, logic reasoning, digital hardware synthesis and beyond.
- Theorems: all proofs are derived from the four postulates.
- Transformations: Boolean algebra for the hardware designs (cost and performance).
- Switching function: truth table, K map (CSE140)