CSE 20
Discrete Mathematics

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http://cseweb.ucsd.edu/classes/sp12/cse20-a/

Textbooks

- **A Short Course in Discrete Mathematics**
  - Edward A. Bender and S. Gill Williamson
  - http://cseweb.ucsd.edu/~gill/BWLectSite/
  - Hardcopy published by Dover, 2004
- **Discrete Mathematics**
  - Seymour Lipschutz and Marc Lipson
  - Schaum's Outline Series, Third Edition, McGraw Hill, 2009

Grading

- iCliker (ramp function saturates at 80% clicks) 7%
- Discussion Session Attendance 3%
- CK Office Hrs Visits 2%
- Midterm 1 25%

  Th 4/19/2012
- Midterm 2 25%

  Th 5/10/2012
- Final Exam 40%
  - (comprehensive with emphasis on the contents after Midterm 2)

  M 6/11/2012, 3-5:59PM

Expectation

- Class participation and group discussion
- Discussion session attendance
- Office hour visits
- Class notes
- Exercises

Administrative

- **Schedule**
  - Lectures: 3:30-4:50PM TTh, Center 214.
  - Discussion:
    - 2:00-2:50PM M, Center 109.
    - TBA F, TBA
    - First Discussion Section: 4/9
  - CK Cheng Office Hours: CSE2130
    - 2:00-2:50PM T,
    - 11:00-11:50AM Th

Course Outline

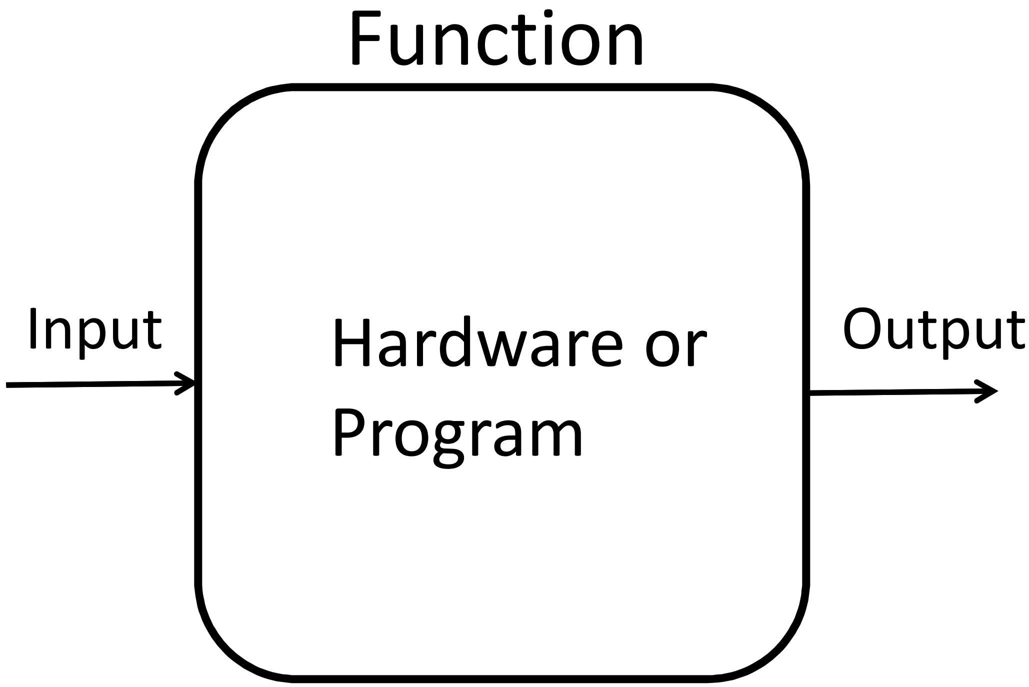
Part 1. Numbers: choice of number systems, binary, Gray code, one's complement, two's complement, residual number system, and coding.
Part 2. Boolean Algebra: manipulation of logic and gates, laws and theorems, tautology, SAT, multiple elements, minimization.
Part 3. Functions and Recursion: function definition and calculation, induction process, k'th order series, Factorial, Fibonacci, Ackerman, division, square root iterations.

Overall View

Numbers, Texts

Images

Control signals



Goal: Cost, Performance, Power, Reliability

Arithmetic: +,-,x,/

Logic: AND, OR, NOT

Permutation: Ordering

Part I. Number Systems

1. Introduction (Why binary system?)
2. Binary Number B.F. Section 2
3. Gray Code (Variations of binary system)
4. Negative Numbers B.F. Section 2 (Hardware implementation)
5. Residual Numbers N.T. Section 1, Shaum Ch. 11
6. Cryptography N.T. Section 2

I. Introduction (Why binary number?)

1. Numbers in general
2. Radix number systems
3. Efficiency of the systems
4. Remarks

1.1 iClicker

Usage of number systems for computers is:

- A. to represent a set of numbers
- B. to provide a unique index for every object
- C. to reflect the algebraic and arithmetic structure of the numbers
- D. All of the above.

1.1 Numbers in General

Symbols and Positions

- Roman numeral
  - Symbols: I, V, X, L, C, D, M
  - Positions: I, II, III, IV, V, VI, …, IX, X, XI,
- Time
  - Symbols: 0-11 month, 0-29 day, 0-23 hour, 0-59 min, 0-59 second
  - Positions: e.g. 3 hrs 45 minutes
- Arabic numeral
  - Symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - Positions: 1, 2, …, 9, 10, 11, 12, …, 20, 21

1.2 Radix Number Systems

- Decimal number: 0123456789
  - E.g. $(1026)_{10}$
- Binary number: 01
  - E.g. $(10000000010)_2$
- Octal number: 01234567
  - E.g. $(2002)_8$
- Hexadecimal: 0123456789ABCDEF
  - E.g. $(402)_{16}$
- Hybrid radix number
  - Varies on weights of the positions and range of symbols per position
  - Example: time

1.2 Radix Number Systems

Definition: A number system of radix r and n digits uses the format:

$(b_{n-1}, \ldots, b_1, b_0)_r$

where $0 <= b_i < r$ for $0 <= i < n$.

Value: $b_{n-1}r^{n-1} + \ldots + b_1 r^1 + b_0 r^0$

Range: r^n $[0, r^n - 1]$

tokens: rxn

1.2 Radix Number Systems

- Decimal (radix r=10)
  - Each digit belongs to the set {0,1,2,3,4,5,6,7,8,9}
  - Example: $(250)_{10}=2*10^2 \quad + \quad 5*10$
  - An n digit decimal number system covers 10^n numbers from 0 to 10^n-1
- Binary (radix r= 2)
  - Each digit belongs to the set {0,1}
  - Example: $(10111)_2= 2^4 +2^2 + 2^1 + 2^0 = 23$
  - An n digit binary number system covers 2^n numbers from 0 to 2^n-1
- Ternary (radix r=3)
  - Each digit belongs to the set {0,1,2}
  - Example: $(1202)_3= 3^3+2x3^2+2x3^0 =27+18+2=47$
  - An n digit ternary number system covers 3^n numbers from 0 to 3^n-1

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The value of a binary number $(1011)_2$ is

- A. 3
- B. 7
- C. 11
- D. None of the above

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The value of a ternary number $(211)_3$ is

- A. 3
- B. 4
- C. 22
- D. None of the above

1.3 Efficiency of Number Systems

Efficiency: #tokens vs. range of the numbers

- Binary (r=2): With n digits, we use 2n tokens to represent 2^n numbers
- Ternary (r=3): With n digits, we use 3n tokens to represent 3^n numbers
- Octal (r=8): With n digits, we use 8n tokens to represent 8^n numbers
- Decimal (r=10): With n digits, we use 10n tokens to represent 10^n numbers

1.3 Efficiency of Number Systems

Given 30 tokens, how many numbers can we represent?

- Binary: The length of the number n=15 (2n=30).
  - 2^{15} =~33,000
- Ternary: The length of the number is n=10 (3n=30).
  - 3^{10} =~60,000
- Radix 5: The length of the number is n=6 (5n=30).
  - 5^{6} =~16,000
- Decimal: The length of the number is n=3 (10n=30).
  - 10^{3} =~1,000

Which is The Most Expressive?

Given radix r with n digits, #tokens t= r x n

- range of the numbers: $r^n = r^{t/r}$
- We fix t to maximize the range

$$\max_r r^{t/r}$$

- In real space, the solution is r=e (2.718…)
- In VLSI technology, binary is a convenient choice.
  - Switch (off, on) or Voltage (0, Vdd)

1.4 Remarks

- We design number systems according to the usages and technologies.
- For VLSI designs, binary number system is consistent with the technology.
- Various number systems are possible for different goals and technologies, e.g. low power, reliability, security, bandwidth.

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The range of a binary number system with 32 digits is around

- A. 4×10^6
- B. 10^9
- C. 4×10^9
- D. 4×10^{12}