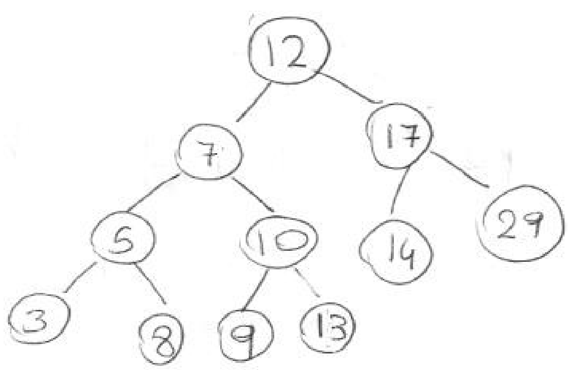
DISCUSSION OF FRIDAY 25th: RECURSION

BINARY SEARCH TREES (RECURSION)

Given a set of elements, we want to order them quickly and efficiently

Example: $L = \{14, 12, 8, 3, 17, 13, 7, 29, 5, 10, 9\}$

① Take any elem as a "root". E.g: 12
Each node has max 2 "children". On the left, one that is smaller, on the right one that is bigger than the node itself



- A bin. tree is SORTED if every node in the tree is larger than (or equal to) its left child, and smaller than (or equal to) its right child
- Equal nodes can go either on the right or on the left (but it has to be consistent)
- Searching a binary tree (to find a value) can be done recursively ("tree trasversal").
You can code a bin. tree through recursion.

EXAMPLE RECURSIVE FUNCTION

$$\begin{cases} F(0) = 0 \\ F(1) = 1 \\ F(n) = \left(F(n-1)\right)^{n} + \left(F(n-2)\right)^{-(n-1)} & , n > 1 \end{cases}$$

? $\rightarrow$ $F(5)$

$F(2) = 1^{2} + 0^{-1} = 1$

$F(3) = 1^{3} + 1^{-2} = 2$

$F(4) = 2^{4} + 1^{-3} = 2^{4} + 1 = 17$

$F(5) = (17)^{5} + (2)^{-4} = 17^{5} + \frac{1}{2^{4}} \doteq 83,521.0625$

The Fibonacci Sequence

•"Facts" about rabbits
– Rabbits never die
– A rabbit reaches maturity exactly two months after birth, that is, at the beginning of its third month of life
– Rabbits are always born in male-female pairs

• At the beginning of every month, each mature male-female pair gives birth to exactly one male-female pair

Problem **How many pairs of rabbits are alive in month n?**

Solution:
Recurrence relation: rabbit(n) = rabbit(n-1) + rabbit(n-2)

Base cases
rabbit(2), rabbit(1)
Recursive definition

$$\text{rabbit}(n) = \begin{cases} 1 & \text{if } n \text{ is 1 or 2} \\ \text{rabbit}(n-1) + \text{rabbit}(n-2) & \text{if } n > 2 \end{cases}$$

Fibonacci sequence: The series of numbers rabbit(1), rabbit(2), rabbit(3), and so on

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Example: Consider the recurrence relation $a_n = 2a_{n-1} – a_{n-2}$ for n = 2, 3, 4, …

- Is the sequence $\{a_n\}$ with $a_n=3n$ a solution of this recurrence relation?

Solution:
If $a_n=3n$ then it must be that: $a_{n-1}=3(n-1)$ and $a_{n-2}=3(n-2)$

For n >= 2 we see that $2a_{n-1} – a_{n-2} = 2(3(n – 1)) – 3(n – 2) = 3n = a_n$.

Therefore, $\{a_n\}$ with $a_n=3n$ is a solution of the recurrence relation.

- Is the sequence $\{a_n\}$ with $a_n=5$ a solution of the same recurrence relation?

Solution:

For n>= 2 we see that $2a_{n-1} – a_{n-2} = 2*5 - 5 = 5 = a_n$.

Therefore, $\{a_n\}$ with $a_n=5$ is also a solution of the recurrence relation.