

1 Please Notice

- Only final answers available.
- Solutions and proofs not included.
- Proofs for recursive sequences please refer to questions 6.34 and 6.35 in Shaum's series.
- For each sequence, the RANK for the first number is 0.

2 Binary Number Systems

1.

$$-10 \Rightarrow 001010 \Rightarrow 110101$$

$$-5 \Rightarrow 000101 \Rightarrow 111010$$

$$(-10) + (-5) = 110101 + 111010 = 101111 + 1 = 110000$$

2.

$$\text{Definition } -x \Rightarrow 10^n - x$$

$$\text{Range } [-5 \times 10^{n-1}, 5 \times 10^{n-1} - 1]$$

$$\text{Arithmetic } -216_{10} + 65_{10} = 10^6 - 216 + 65 = 999849$$

3.

$$\text{Definition } -x \Rightarrow 8^n - x$$

$$\text{Range } [-4 \times 8^{n-1}, 4 \times 8^{n-1} - 1]$$

$$\text{Arithmetic } -120_8 - 27_8 =$$

$$8^6 - 120_8 + 8^6 - 27_8 =$$

$$777660_8 + 777751_8 =$$

$$1777631_8 =$$

$$777631_8^1$$

3 Boolean Algebra

$$2. ab' + bc + a'c'$$

¹As we have only 6-digit, the highest is then filtered out.

3. $x'y'z'$

4. $x + y$

5. $ab + b'd' + c' + ad$

6. $x' + y + z'$

4 Recursive Function

4.1 Recursive function: formulation

1.

(a) 591

(b) 324561

2.

(a)

$$H(6, S, E, G) \Rightarrow$$

$$H(5, E, S, G) \Rightarrow$$

$$H(4, S, E, G) \Rightarrow$$

$$H(3, S, G, E) \Rightarrow$$

$$H(2, G, S, E) \Rightarrow$$

$$G \xrightarrow{2} E$$

(b)

$$G \xrightarrow{2} E$$

$$S(4, 1), E(3), G(6, 5, 2)$$

(c)

$$G \xrightarrow{1} S$$

$$S(4), E(3), G(6, 5, 2, 1)$$

(d) $S \xrightarrow{1} E$

(e) 53

3.

(a) 101011, 101110

(b) 110000

(c) 42

(d) 110111

4.2 Recursive function: induction

4.3 Recursive function: analysis

1. (a) $a_1 = 2, a_2 = 7, a_3 = 20$

(b) $a_n = 2 \times a_{n-1} + 3 \times a_{n-2}$

(c) $a_n = \frac{3}{4} \times 3^n + \frac{1}{4} \times (-1)^n$

2. $a_n = \frac{1}{4} + (-\frac{1}{4}) \times (-1)^n + \frac{1}{2} \times n \times (-1)^n$