CSE20 Midterm 2, May 12, 2011, Name______________________

1. Residual Number System (10 points): Show the operation of 8×17 in a residual number system with moduli $(m_1, m_2, m_3) = (3, 7, 8)$.

2. Residual Number System (15 points): Suppose $(x\%4, x\%5, x\%7) = (1, 2, 3)$, where symbol % denotes modulus operation. Follow the procedure of Chinese remainder theorem to derive the smallest positive integer x that satisfies this system.

3. Boolean Algebra (10 points): State the definition of Boolean algebra.

4. Boolean Algebra (10 points): Use Boolean algebra (laws and theorems) to prove the De Morgan's theorem: $(ab)' = a' + b'$.

5. Boolean Algebra (15 points): Use Boolean algebra (laws and theorems) to transform Boolean function, $E(a, b, c) = a'bc + ab' + bc'$, into product-of-sums form.

6. Boolean Algebra (15 points): Reduce the following to an expression of a minimal number of literals: $E(a, b, c) = a'b' + b'c + ac + bc + a'bc'$.

7. Recursive Function (15 points): A frog knows 3 jumping styles (A, B, C). Styles A, B jump forward by 1 foot, and style C jumps forward by 2 feet. Let a_i denote the number of ways to jump over a total distance of i feet.

(a) What is a_1, a_2, a_3?
(b) Derive the recursive formula of a_n.
(c) Find the solution of the recursion.

8. Recursive Function (10 points): Consider the following homogeneous linear recurrence relation: $a_n = 2ra_{n-1} - r^2 a_{n-2}$. Show that $a_n = c_1 r^n + c_2 n r^n$ satisfies the recurrence relation, where c_1, and c_2, are constant coefficients.