

1. Convert the following as indicated.
 - 1a. Convert octal 102_8 to decimal (5 points).
 - 1b. Convert $B7C5_{16}$ to octal (i.e. base 8) (5 points).
2. Find the range of the following number systems.
 - 2a. A hybrid system has 2 digits of radix 3 and 7 digits of radix 5. Describe the range of the system (5 points).
 - 2b. A two's complement system has 10 bits. Describe the largest and smallest numbers (5 points).
3. Write the truth table of a full adder with three binary inputs a, b, c_{in} and two binary outputs c_{out}, s_{um} . Note that in arithmetic, we have $a + b + c_{in} = 2c_{out} + s_{um}$ (10 points).
4. Write the sequence of a 4-bit Gray code (10 points).
5. Given $x = 17_{10}$ and $y = 12_{10}$, show the operation of $-x - y$ in 9-bit one's complement representation. Write the answer with exactly 9 bits (15 points).
6. We have defined and learned the idea of two's complement for n-bit binary numbers. Given an n-digit system with base 8, define the eight's complement representation. Show the arithmetic of $-x + y$ where $x = 216_8$ and $y = 65_8$ with a 7-digit system in eight's complement representation (15 points).
7. Find x in the following equations, where symbol % denotes modulus operation.
 - 7a. $(18)\%7 = x$ (5 points).
 - 7b. $(-25)\%7 = x$ (5 points).
 - 7c. $(9x)\%7 = 1$ and $0 \leq x < 7$ (5 points).
 - 7d. $(-8x)\%7 = 1$ and $0 \leq x < 7$ (5 points).
8. Given three integers x, y, d , prove that $(x\%d \times y\%d)\%d = (x \times y)\%d$, where % is a modulus operation (10 points).