CSE20 Final Exam, June 11, 2012, Name_____

1. (number systems: one's complement) Show the operation of $17 + (-14)$ in one's complement of binary number system. Assume that each binary number is represented with 10 bits. (10 points)

2. (number systems: two's complement) We have defined and learned the idea of two's complement for n-bit binary numbers.

2.1. Define the complement (corresponding to two's) using an n-digit system with base 8. (5 points)

2.2. Show the arithmetic of $x - y$ where $x = 11_8$ and $y = 17_8$ in the complement representations (corresponding to two's) using a 5-digit system with base 8. (5 points)

3. (Boolean algebra: proof of consensus theorem) Prove the following equality using Boolean algebra laws and theorems.

3.1 Prove the consensus theorem: $ab + a'c = ab + a'c + bc$. (5 points)

3.2 Prove the Boolean equality $(a + b)(a' + c) = (a + b)(a' + c)(b + c)$. (5 points)

4. (Boolean algebra) Express Boolean function $E(a, b, c) = (b + ac')'(ab' + c)$ in sum-of-products form using Boolean algebra laws and theorems. Express in the minimal expression. (10 points)

5. (Boolean algebra: product of sums) Express Boolean function $E(a, b, c) = a'bc + b'[(a+b')(a+c)]'$ in product-of-sums form using Boolean algebra laws and theorems. Express in the minimal expression. (10 points)

6. (recursive function: permutation) Suppose all the permutations on the set of $\{1, 2, 3, 4, 5, 6\}$ are listed in lexicographic order from 0 to $6! - 1$.

6.1 What is the RANK (order) in the list for 453261? (10 points)

6.2 What permutation will have the RANK 165? (5 points)

7. (recursive function: induction) Use induction to prove the following identity for any positive integer n: $1 \times 2 + 2 \times 3 + ... + (n-1) \times n = n(n-1)(n+1)/3$. (10 points)

8. (recursive function: induction) Prove by induction that any postage of at least 8 cents can be obtained using 3 cents and 5 cents stamps. (10 points)

9. (recursive function) A frog knows 3 jumping styles (A, B, C). With style A the frog jumps forward by 1 feet, and with styles, B, C, the frog jumps forward by 2 feet. Let a_i denote the number of ways to jump over a total distance of i feet.

9.1. Write the values of a_1, a_2, a_3? (5 points)

9.2. Derive the recursive formula of a_n? (5 points)

9.3. Find the solution of the recursion. (5 points)