

Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 3

CSE152, Spr 2012

Intro Computer Vision

Announcements

- Assignment 0: due Thursday 4/12
- Sign up for Piazza: <http://www.piazza.com>
- Office Hour: Tuesdays 1:00-2:00
- Read Trucco & Verri: pp. 15-40,
- Optional Reading, Szeliski, Chapter 2

CSE152, Spr 2012

Intro Computer Vision

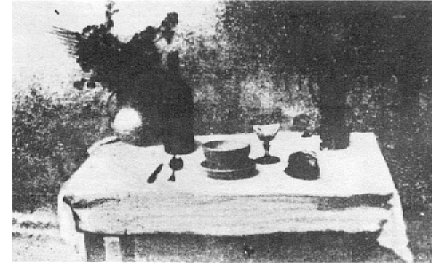
Image Formation: Outline

- Factors in producing images
- Projection
- Perspective
- Vanishing points
- Orthographic
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance

CSE152, Spr 2012

Intro Computer Vision

Earliest Surviving Photograph



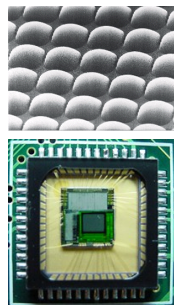
- First photograph on record, “la table service” by Nicéphore Niépce in 1822.
- Note: First photograph by Niépce was in 1816.

CSE152, Spr 2012

Intro Computer Vision

How Cameras Produce Images

- Basic process:
 - photons hit a detector
 - the detector becomes charged
 - the charge is read out as brightness
- Sensor types:
 - CCD (charge-coupled device)
 - high sensitivity
 - high power
 - cannot be individually addressed
 - blooming
 - CMOS
 - most common
 - simple to fabricate (cheap)
 - lower sensitivity, lower power
 - can be individually addressed



CSE152, Spr 2012

Intro Computer Vision

Images are two-dimensional patterns of brightness values.

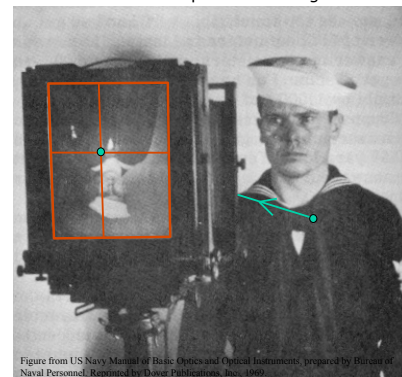


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

CSE152, Spr 2012

They are formed by the projection of 3D objects

Effect of Lighting: Monet



CSE152, Spr 2012

Intro Computer Vision

Change of Viewpoint: Monet



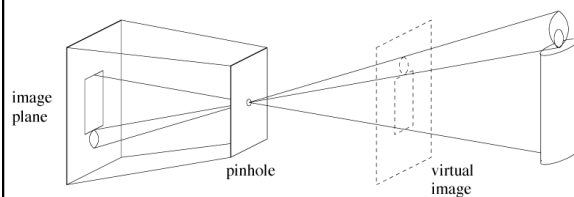
Haystack at Chailly at Sunrise (1865)

CSE152, Spr 2012

Intro Computer Vision

Pinhole Camera: **Perspective projection**

- Abstract camera model - box with a small hole in it

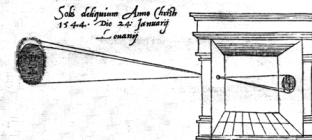


CSE152, Spr 2012 Forsyth&Ponce

Intro Computer Vision

Camera Obscura

illum in tabula per radios Solis, quum in caelo contingit: hoc effusi in caelo superior pars deliquit patitur, in radius apparebit inferior deficere, vt ratio exigit optica.



Sic nos exatit Anno .1544. Louani eclipsum Solis obseruauimus, inuenimusq; deficere paulo plus q; dex.

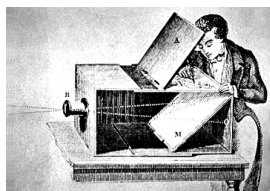
"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

Leonardo da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

CSE152, Spr 2012

Intro Computer Vision



- Used to observe eclipses (eg., Bacon, 1214-1294)
- By artists (eg., Vermeer).

CSE152, Spr 2012

Intro Computer Vision

Camera Obscura



Jetty at Margate England, 1898.

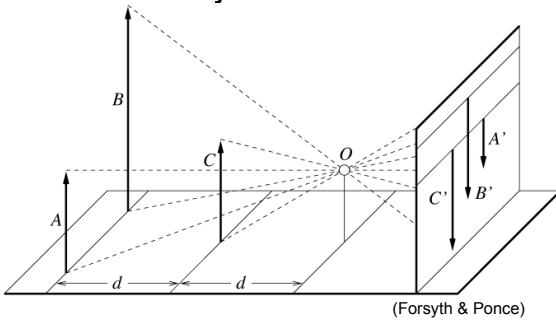


<http://brightbytes.com/cosite/collection2.html> (Jack and Beverly Wilgus)

CSE152, Spr 2012

Intro Computer Vision

Distant objects are smaller



(Forsyth & Ponce)

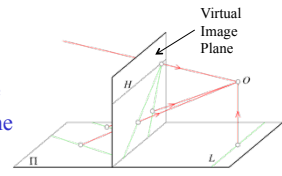
Note “intersection of rays” ala Leonardo.

CSE152, Spr 2012

Intro Computer Vision

Geometric properties of projection

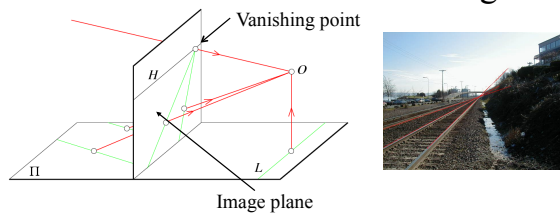
- Points go to **points**
- Lines go to **lines**
- Planes go to **whole image or half-plane**
- Polygons go to **polygons**
- Angles & distances not preserved
- Degenerate cases:
 - line through focal point yields **point**
 - plane through focal point yields **line**



CSE152, Spr 2012

Intro Computer Vision

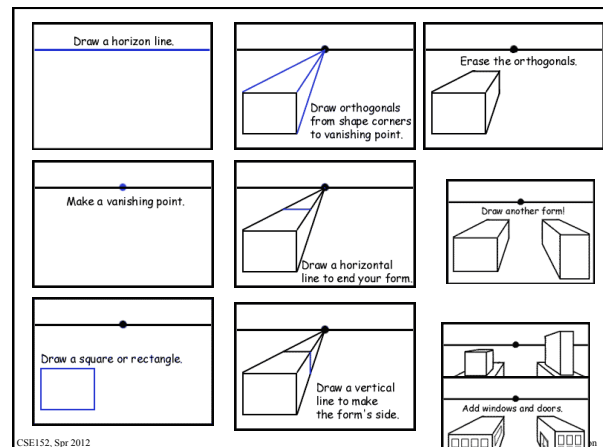
Parallel lines meet in the image



- The projection of parallel lines meet at the vanishing point
- Intersection of line through O parallel to the 3-D line(s)
- A single line can have a vanishing point

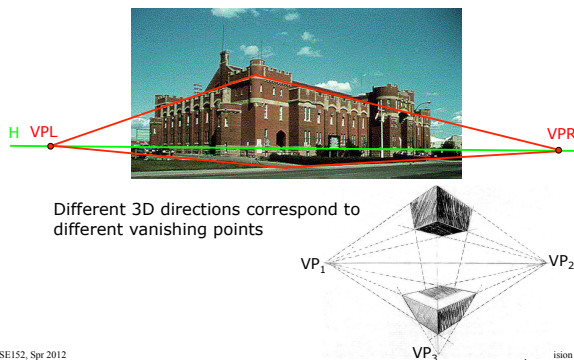
CSE152, Spr 2012

Intro Computer Vision



CSE152, Spr 2012

Vanishing points

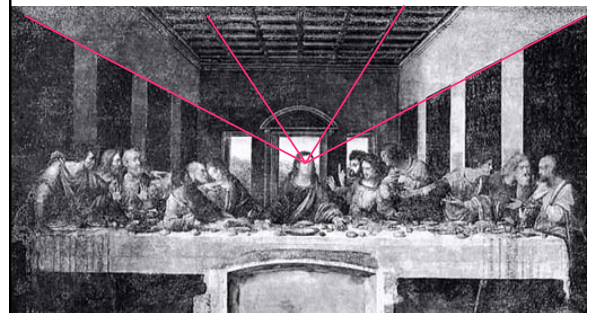


Different 3D directions correspond to different vanishing points

CSE152, Spr 2012

ision

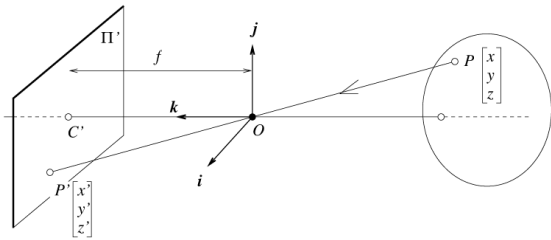
Vanishing Points



CSE152, Spr 2012

Intro Computer Vision

The equation of projection



Cartesian coordinates:

- We have, by similar triangles, that $(x, y, z) \rightarrow (f x/z, f y/z, -f)$
- Ignoring the third coordinate, we get

$$(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z}\right)$$

CSE152, Spr 2012

Intro Computer Vision

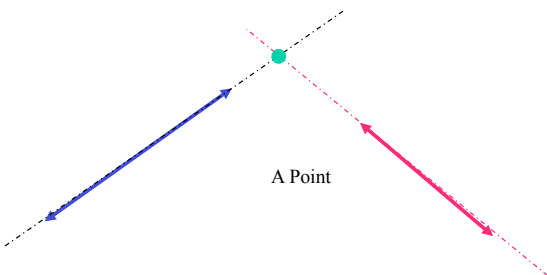
A Digression

Homogenous Coordinates
and
Camera Matrices

CSE152, Spr 2012

Intro Computer Vision

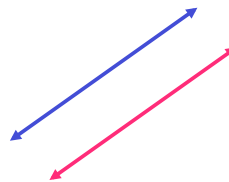
What is the intersection of
two lines in a plane?



CSE152, Spr 2012

Intro Computer Vision

Do two lines in the plane always
intersect at a point?



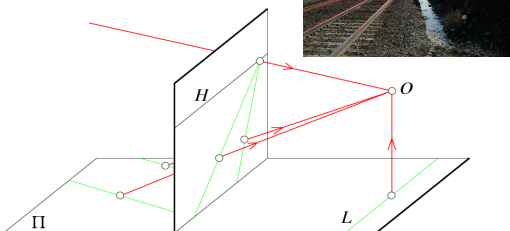
No, Parallel lines don't
meet at a point.

CSE152, Spr 2012

Intro Computer Vision

Can the perspective image of two parallel
lines meet at a point?

YES



CSE152, Spr 2012

Intro Computer Vision

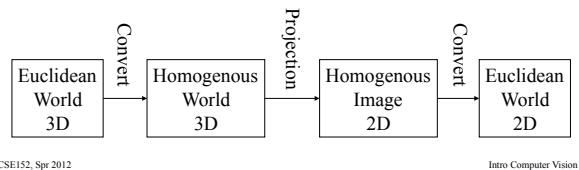
Projective geometry provides an elegant
means for handling these different
situations in a unified way and
homogenous coordinates are a way to
represent entities (points & lines) in
projective spaces.

CSE152, Spr 2012

Intro Computer Vision

Homogenous coordinates

- Our usual coordinate system is called a Euclidean or affine coordinate system
- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies



Homogenous coordinates

A way to represent points in a projective space

1. Add an extra coordinate
e.g., $(x,y) \rightarrow (x,y,1) = (u,v,w)$
2. Impose equivalence relation such that $(\lambda \text{ not } 0)$
 $(u,v,w) \approx \lambda * (u,v,w)$
i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$
3. "Point at infinity" – zero for last coordinate
e.g., $(x,y,0)$

- Why do this?
 - Possible to write the action of a perspective camera as a matrix
 - Possible to represent points "at infinity"
 - Where parallel lines intersect
 - Vanishing points are the projection of points of points at infinity

CSE152, Spr 2012

Intro Computer Vision

Changes of coordinates: Euclidean $\rightarrow$ Homogenous $\rightarrow$ Euclidean

In 2-D

- Euclidean $\rightarrow$ Homogenous: $(x, y) \rightarrow k(x, y, 1)$
- Homogenous $\rightarrow$ Euclidean: $(u, v, w) \rightarrow (u/w, v/w)$

In 3-D

- Euclidean $\rightarrow$ Homogenous: $(x, y, z) \rightarrow k(x, y, z, 1)$
- Homogenous $\rightarrow$ Euclidean: $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

CSE152, Spr 2012

Intro Computer Vision

The camera matrix

$$(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z} \right)$$

Turn this expression into homogenous coordinates

- HC's for 3D point are (X, Y, Z, T)
- HC's for point in image are (U, V, W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Perspective
Camera Matrix
A 3x4 matrix

CSE152, Spr 2012

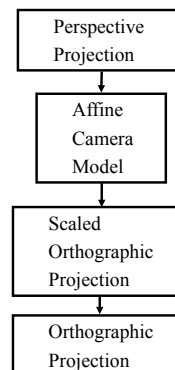
Intro Computer Vision

End of the Digression

CSE152, Spr 2012

Intro Computer Vision

Simplified Camera Models



Affine Camera Model

Appropriate
in Neighborhood
About (x_0, y_0, z_0)

- Take Perspective projection equation, and perform Taylor Series Expansion about some point (x_0, y_0, z_0) .
- Drop terms of higher order than linear.
- Resulting expression is the affine camera model

CSE152, Spr 2012 Intro Computer Vision

- Perspective

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z} \begin{bmatrix} x \\ y \end{bmatrix}$$
- Assume that $f=1$, and perform a Taylor series expansion about (x_0, y_0, z_0)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \frac{1}{z_0^2} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0) + \frac{1}{z_0} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (x - x_0) + \frac{1}{z_0} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (y - y_0) + \frac{1}{2} \frac{2}{z_0^3} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0)^2 + \dots$$
- Dropping higher order terms and regrouping.

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}\mathbf{p} + \mathbf{b}$$

CSE152, Spr 2012 Intro Computer Vision

This is called the Affine Camera Model

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}\mathbf{p} + \mathbf{b}$$

Rewrite affine camera model in terms of homogenous coordinates

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\ 0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Affine Camera Matrix

CSE152, Spr 2012 Intro Computer Vision

Orthographic Camera Model

Consider doing an expansion about a point along the optical axis $X_0 = 0, Y_0 = 0$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\ 0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/z_0 & 0 & 0 & 0 \\ 0 & 1/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Orthographic Camera Matrix

Unlike perspective,

- Parallel lines project to parallel lines
- Ratios of distances are preserved under orthographic

CSE152, Spr 2012 Intro Computer Vision

Orthographic projection

Starting with affine camera model
Take Taylor series about $(0, 0, z_0)$ – a point on optical axis

Focal length=1 Focal length=f

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z_0} \begin{bmatrix} x \\ y \end{bmatrix}$$

CSE152, Spr 2012 Intro Computer Vision

Other camera models

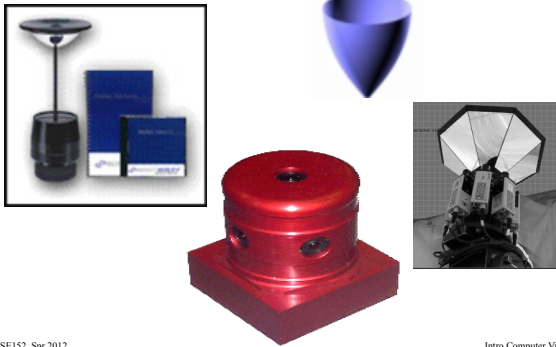
- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omniscam (hemispherical)

Light Probe (spherical)

CSE152, Spr 2012 Intro Computer Vision

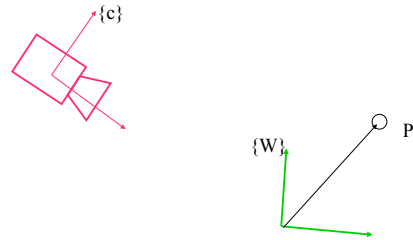
Some Alternative “Cameras”



CSE152, Spr 2012

Intro Computer Vision

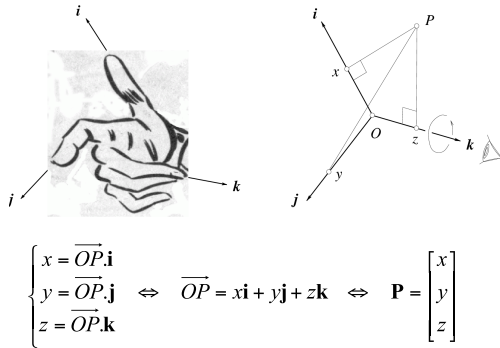
What if camera coordinate system differs from object or world coordinate system



CSE152, Spr 2012

Intro Computer Vision

Euclidean Coordinate Systems

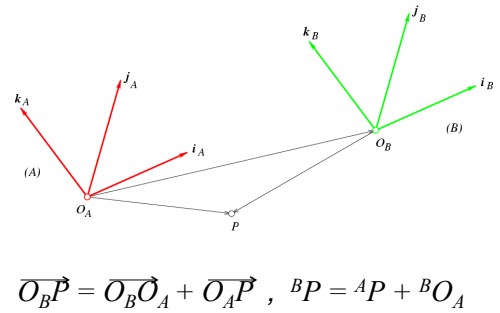


CSE152, Spr 2012

Intro Computer Vision

Coordinate Changes: Pure Translations

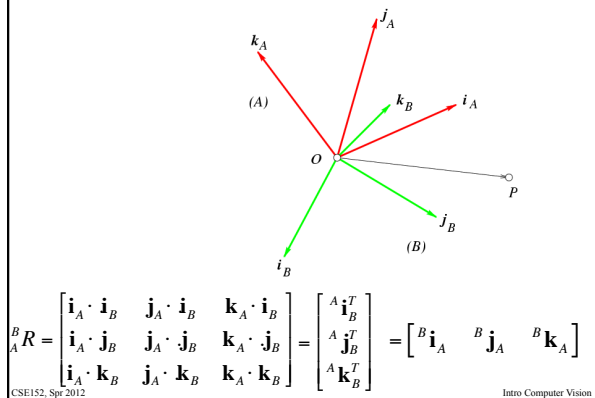
No rotation (e.g., $\mathbf{i}_A = \mathbf{i}_B$ etc)



CSE152, Spr 2012

Intro Computer Vision

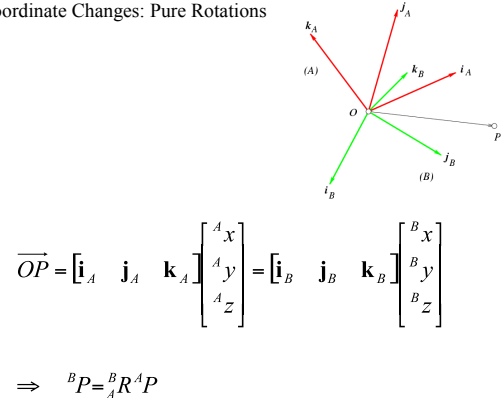
Rotation Matrix



CSE152, Spr 2012

Intro Computer Vision

Coordinate Changes: Pure Rotations



CSE152, Spr 2012

Intro Computer Vision

A rotation matrix $\mathbf{R}$ has the following properties:

- Its inverse is equal to its transpose $\mathbf{R}^{-1} = \mathbf{R}^T$
- Its determinant is equal to 1: $\det(\mathbf{R})=1$.

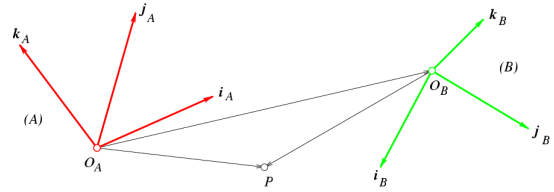
Or equivalently:

- Rows (or columns) of $\mathbf{R}$ form a right-handed orthonormal coordinate system.

CSE152, Spr 2012

Intro Computer Vision

Coordinate Changes: Rigid Transformations



$${}^B P = {}^B R {}^A P + {}^B O_A$$

CSE152, Spr 2012

Intro Computer Vision