

CSE 101 Homework 2
Greedy Algorithms
Due Tuesday, May 3

Directions: For each of the first four problems, a "high level" greedy strategy is given. For some of the problems, the strategies give a correct (optimal) solution, and for others, it sometimes gives incorrect (suboptimal) solutions. For each, decide whether the greedy strategy produces optimal solutions. If it is, give a proof that it is correct, then describe what data structures and preprocessing you would use to give an efficient version, and give a time analysis. (10 points, correctness proof, 10 points efficiency and correct time analysis)

If it is not correct, give a counter-example showing the strategy is incorrect. Then still give an efficient version, as a heuristic. (10 points, counter-example, 10 points efficiency and correct time analysis)

Caravan stops You are organizing a caravan crossing a desert. Your path is fixed. The caravan can only travel m miles between stops at oases. You are given a list of oases $Oasis[1..n]$, each with its distance d_i in miles from the starting point. The last oasis, O_n , is the destination. You wish to choose the minimum size set of stops subject to the constraint that there are no more than m miles between consecutive stops.

Candidate greedy strategy: Treat the start as an oasis with $d_i = 0$. At each stop, at oasis $Oasis[i]$, if the destination $d_n \leq d_i + m$, go there directly. Otherwise, find the oasis j of maximum d_j subject to $d_j \leq d_i + m$. Make j your next stop, and repeat.

Maximum independent set An *independent set* in an undirected graph $G = (V, E)$ is a set of nodes $S \subseteq V$, so that no two nodes in S are adjacent in E . i.e., if $\{x, y\} \in E$, we cannot have both x and y in S . The maximum independent set problem is to find a largest independent set in a given graph.

Candidate greedy strategy: Pick the node x of smallest degree, and put $x \in S$. Remove x and all of its neighbors from G . Repeat until no nodes are left.

Oxen pairing Consider the following problem: We have n oxen, $Ox_1, \dots, Ox_n$, each with a strength rating S_i . We need to pair the oxen up into teams to pull a plow; if Ox_i and Ox_j are in a team, we must have $S_i + S_j \geq P$, where P is the weight of a plow. Each ox can only be in at most one team. Each team has exactly two oxen. We want to maximize the number of teams.

Candidate Greedy Strategy: Take the strongest and weakest oxen. If together they meet the strength requirement, make them a team. Recursively find the most teams among the remaining oxen.

Otherwise, delete the weakest ox. Recursively find the most teams among the remaining oxen.

Cookie assignment Consider the following problem:

You are baby-sitting n children and have $m > n$ cookies to divide between them. You must give each child exactly one cookie (of course, you cannot give the same cookie to two different children). Each child has a greed factor $g_i, 1 \leq i \leq n$ which is the minimum size of a cookie that the child will be content with; and each cookie has a size $s_j, 1 \leq j \leq m$. Your goal is to maximize the number of content children, i.e., children i assigned a cookie j with $g_i \leq s_j$.

Candidate Strategy : Look at the greediest child. If the largest cookie makes the child content, give the child the largest cookie. Otherwise, give the child the smallest cookie.

Implementation Implement the greedy algorithm for maximum independent set and test it on random graphs where each possible edge is in the graph with probability $1/2$. What is the average size of the independent set it finds for graphs of different sizes? (Try n as many powers of 2 as you can.) How do you conjecture the size will grow as a function of n ?