

## CSE252B – Computer Vision II – Assignment #3

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<http://www-cse.ucsd.edu/classes/sp10/cse252b>

Due Date: Thurs. May. 13, 2010.

1. Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola or parabola.
2. Consider the Lyapunov map from MaSKS Equation (6.42), p. 195:

$$L : \mathbb{C}^{3 \times 3} \rightarrow \mathbb{C}^{3 \times 3}; \quad X \mapsto X - CXC^\top$$

Assume  $C$  has  $n$  independent eigenvectors  $\{\mathbf{u}_i \in \mathbb{C}^n\}_{i=1}^n$ , with eigenvalues given by  $C\mathbf{u}_i = \lambda_i\mathbf{u}_i$ .

- (a) Show that  $X_{ij} = \mathbf{u}_i\mathbf{u}_j^* \in \mathbb{C}^{3 \times 3}$  is an eigenvector of  $L$ . What is the corresponding eigenvalue?
  - (b) Assuming  $\det(C) = 1$ , which eigenvectors are in  $SRker(L)$ ? In other words, for which values of  $i$  and  $j$  does  $L$  map the symmetric real  $X$  to the zero matrix?
  - (c) Explain the significance of  $SRker(L)$  if we interpret  $X$  as the coefficient matrix for a conic.
3. 2D Upgrade from Affine to Euclidean via Orthogonal Lines.
    - (a) Load in the affine-rectified image `affine_tile.gif`, identify two pairs of imaged orthogonal lines, and plot them on the raw image.
    - (b) Implement the algorithm to solve for  $K \in SL(2)/SO(2)$  from two imaged right angles on a plane as described in H&Z Example 2.26 (Metric Rectification I), p. 56.
    - (c) Demonstrate your code on the tile image. Display the Euclidean-rectified image and plot the transformed line pairs on it.
  4. Implement the algorithm described in H&Z Example 8.18 (A Simple Calibration Device), p. 211. Use it to estimate  $K$  from the image `squares.gif` depicting three metric planes. Compare your  $K$  to the  $K$  from H&Z.
  5. Derive the solution for the homography  $H \in GL(4)$  relating  $n \geq 5$  corresponding 3D points  $(\mathbf{X}_1^j, \mathbf{X}_2^j)$ ,  $j = 1, 2, \dots, n$ , in general position.
  6. Uncalibrated 3D Reconstruction.
    - (a) Run the script `house_views.m` to produce two uncalibrated views of a wireframe house.
    - (b) Recover  $F$  using the 8-pt. algorithm and compute the canonical camera matrices  $(\Pi_{1p}, \Pi_{2p})$ .
    - (c) Triangulate to produce the projective structure  $\mathbf{X}_p$ .
    - (d) Find the homography  $H \in GL(4)$  to upgrade  $\mathbf{X}_p$  directly to the Euclidean structure  $\mathbf{X}_e$  using 5 ground truth points.
    - (e) Solve for the image of the plane at infinity  $(\mathbf{v}^\top, v_4)^\top$  from three vanishing points and use it to upgrade  $\mathbf{X}_p$  to the affine structure  $\mathbf{X}_a$ .