

**CSE 20 Spring 2009
Exam 5**

1. Consider the following relationship R between social clubs at UCSD. Two clubs are related by R if they have a common member, i.e. there is a student who is a member of both. Consider each of the 3 required conditions for an equivalence relation. Are they satisfied for R , i.e. is it an equivalence relation? Give a short explanation for each. (1.5)

i) reflexive

Yes. Not only is at least one member of a club also a member of that club, ALL students in that club are also members of that club. So ARA is true for all clubs.

ii) symmetric

Yes. For clubs A and B , ARB means there exists a student x that is in both A and B . That same student is in both B and A . So if ARB is true, then so is BRA .

iii) transitive

No. For clubs A , B , and C , ARB means there exists a student x that is in both A and B and BRC means that there exists a student y that is both B and C . Students x and y COULD be the same student, but they certainly don't have to be.

For the record, the fact that it wasn't transitive means it wasn't an equivalence relation.

2. Consider the following relationship R between people in a set S .

Two people x and y are related by R , i.e. xRy , if y has the same birthday as x . Consider each of the 3 required conditions for an equivalence relation. Are they satisfied for R , i.e. is it an equivalence relation? Give a short explanation for each. (1.5)

i) reflexive

Yes. A person has the same birthday as him- or herself. So xRx .

ii) symmetric

Yes. If persons x and y have the same birthday, then so do y and x . Basically the relation partitions people up by the day they were born and x and y are in the same block. So if xRy then yRx .

iii) transitive

Yes. Suppose xRy and yRz . Then persons x and y have the same birthday, a , and y and z have the same birthday, b . Because person y can only have one birthday we know $a = b$. So x and z both have birthday a . So given xRy and yRz , it must be the case that xRz .

For the record, that meant R is an equivalence relation.

3. In any group of 677 people, there must be at least two who have the same first and last letters of their names. Explain (1.5)

There are 26 letters in the alphabet.

26 choices for the first letter.

26 choices for the last letter.

There are therefore $26 \times 26 = 676$ possible pairs of letters.

So by the pigeonhole principle, with 677 people 2 people (at least) must share first/last letters.

4. What is the smallest integer k such that any set S of k people must have at least 3 people who were born in the same month of the year. (1.5)

There are 12 months in a year.

So by the pigeonhole principle $12 \times 2 + 1 = 25$ people guarantee at least 3 people born in the same month.

24 would not suffice – then each month could have just 2 people.

5. What are the three properties required for a binary relationship on a set S to be an order relationship (i.e. a partial order). (1.5)

i) Reflexive: xRx

ii) Anti-symmetric: xRy and yRx ONLY if $x=y$.

iii) Transitive: If xRy and yRz , then xRz .

You only had to provide the names.

6. Consider the following set and relationship. (1.5)

$$S = \{a, b, c, d\}$$

$$R = \{(a, b), (b, c), (a, a), (b, b), (c, c), (c, d)\}$$

Is the relation R an order relationship? Explain your answer.

No.

There were actually two reasons you could give.

R does not contain (d, d), violating transitivity.

R contains (a, b) and (b, c) but not (a, c) – this violates transitivity.

7. Let S be a set with n elements. (3)**a) How many binary relations are there on S?**

There are $n \times n$ possible pairs (n choices for the first part, n for the second).

Each pair can either be in R or not in R – a binary choice.

Therefore there are $2^{(n^2)}$ (aka 2 to the n-squared power) binary relations for sets with n elements.

$$2^{n^2}$$

b) How many reflexive relations are there on S?

The idea is similar, but we know that for all x in S, (x, x) is in R. We have to remove the choice that (x, x) isn't in R from the choices we consider. There are n such choices this affects.

So $2^{(n^2 - n)}$

$$2^{n^2 - n}$$

8. Suppose that R is a binary relation on a non-empty set S. (3)

For each x in S, define the set B_x as follows.

$$B_x = \{y \in S, \text{ such that } xRy\}$$

Suppose that R has the following property.

For all x in S, we know that xRx .

Name: _____

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Prove:

i) The sets B_x are not empty, i.e. each set B_x has at least one element.

Since xRx for all x in S , that means $x \in B_x$. Therefore $B_x \neq \emptyset$

In other words, since xRx guarantees that each B_x contains x , B_x can't be empty.

ii) The sets B_x cover S in the sense that each element of S is in at least one B_x .

Since every x in S is in some B_x (namely at least B_x), all elements of S are in at least one B_x .