

CSE 20 Spring 2009
Exam 4

1. The following defines a permutation of $\{1,2,3,4,5\}$ in the 2-line format, and then another permutation in the cycle format. Translate each to the other.

a) Rewrite the following two-line format in cycle form.

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ (& & & &) \\ 2 & 3 & 4 & 5 & 1 \end{array}$$

(1,2,3,4,5)

b) Rewrite the following cycle format in two line format.

(1,4,5) (2) (3)

1 2 3 4 5

4 2 3 5 1

2. Define the set S as follows

$S = \{f \mid f: A \rightarrow B, \text{ where } f \text{ is a one-to-one function}\}$

In each case find $|S|$ (i.e. the size of the set S). Show your work.

a) $|A| = 4$ and $|B| = 4$

Since f is one-to-one, each range object can only be used once.

There are 4 choices for the range object for the first domain object, then 3 for the next, then 2 for the next and then only 1 left for the fourth. So there are $4 \cdot 3 \cdot 2 \cdot 1$ possible mappings.

i.e. 24 possible functions

b) $|A| = 2$ and $|B| = 3$

similarly, there are $3 \cdot 2$ possible mappings

i.e. 6 possible functions

3. Answer the following about the $\in$ (is an element of) and the $\subseteq$ (is a subset of) operators. For each, give True or False.

___ T ___ a) $\{1,2\} \in \{\{1,2\}, \{3,4\}\}$

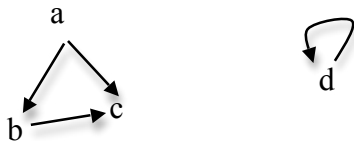
___ F ___ b) $\{2\} \in \{1,2,3,4\}$ The number 2 is an element of the set. The set $\{2\}$ is not.

___ T ___ c) $\{2\} \subseteq \{1,2,3,4\}$

4. Suppose that S is a set. A relation between the elements of S is a subset of $S \times S$. Draw a directed graph diagram of the following relation R on S . (Label the nodes/points in the diagram with the elements of S and put an arrow from x to y if and only if (x,y) belongs to the relation R .)

$$S = \{a,b,c,d\}$$

$$R = \{(a,b), (a,c), (b,c), (d,d)\}$$



5. Is the following a partition of the set $S = \underline{5} = \{1,2,3,4,5\}$. If not, explain.

$$\{\{1,2\}, \{2,3\}, \{4,5\}\}$$

No, because blocks are supposed to be disjoint (no common elements) and the element 2 is in both the first and second subset.

6. Let $P(A)$ stand for the set of all subsets of A , i.e. the power-set of A . Are the following True or False? (If False, give a counter-example).

___ F ___ a) $P(A \cup B) = P(A) \cup P(B)$

Let $A = \{a\}$, and $B = \{b\}$

Then the set $\{a,b\}$ is an element of the powerset on the left, but not on the right.

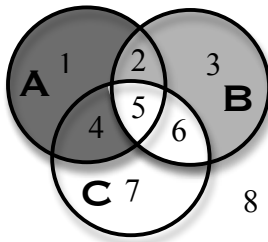
___ T ___ b) $P(A \cap B) = P(A) \cap P(B)$

An element of the set on the right is a subset of both A and B . Hence it is a set whose elements are both in A and in B , hence it is the power set on the left.

An element of the set on the left is a set whose elements are both in A and in B . Hence it is a subset of A and also a subset of B . Hence it is in the powerset on the right.

7. Consider the following. Prove it using Venn diagrams.

$(A-B) \cap (B-C) = \emptyset$, where $\emptyset$ stands for the empty set. The statement asserts that $A-B$ and $B-C$ are disjoint. The notation $X-Y$ means all of X that is not also in Y .



$$A-B = \{1, 4\}$$

$$B-C = \{2, 3\}$$

$$(A-B) \cap (B-C) = \{\}$$
 because there's no overlap.

Comment: To do a proof by Venn diagrams you need to show it's true for the general case (unless you're doing proof by contradiction). This means each ring needs to have areas of overlap and areas without overlap with every other ring. Many student answers had three overlapping rings in a row, with A and C not overlapping – this received half points, as it's showing that it's empty for a particular case but not in general, but was on the right track.

Some had three rings inside each other, covering only regions 1, 2, 5, and 8 in the above diagram. This would mean (for instance) that B and C are both subsets of A . This received zero points. Some also had 3 non-overlapping rings – also not a proof, also zero points.

When doing a proof using Venn diagrams, you need to be clear. You should make sure when you shade different regions that it's clear and have some way of marking where overlap occurs. One good way of doing this was having $A-B$ have lines/shading in one direction and $B-C$ have lines/shading in a different direction and any (potential) areas of overlap have both line orientations. Doing some writing on the side explaining things often helps clarify. A few people lost fractions of a point if it wasn't clear.

8. In each of the following some information is given about a function. (The notation $\underline{n}$ is used to stand for the set $\{1, 2, 3, \dots, n\}$. The notation ^{x} stands for exponentiation, i.e. $x^y = x^y$, and the notation $B^A = B^A$ stands for the set of all functions from A to B .) For each of the functions, is it an injection (1-1), surjection (onto), a bijection (injection and surjection)? Explain your answer.

a) $f \in {}^{\underline{4}}_{\underline{5}}$, $\text{Coimage}(f) = \{\{1, 3, 5\}, \{2, 4\}\}$

This cannot be 1-1 since domain elements 1, 3, and 5 map to the same element in the range, as do domain elements 2 and 3. Hence more than one element in the domain maps to the same element in the range – i.e. it's not the case that each element of the range is

mapped to *at most* once. The fact that the domain was bigger than the range also meant that it simply could not be 1-1.

It cannot be a surjection, because only 2 elements in the range will be mapped to, leaving 2 unused – i.e. it's not the case that each element of the range is mapped to *at least* once. We don't know which range elements are used, but that's ok.

Hence it is not a bijection.

b) $f \in \underline{5}^{\underline{5}}$. Coimage (f) = {{1}, {2}, {3}, {4}, {5}}

It must be 1-1 since each element mapped to by f is only mapped to by one element in the range. In other words, each part of the coimage has only one element in it.

Also, since there are five subsets in the co-image, f must map to 5 different elements in the range, which is all there are. In other words, the size of the coimage and the range were the same. Hence it is a surjection.

Since it's both an injection and a surjection, it is also a bijection.

Some claimed that it was 1-1 because the *domain* and range are the same size. This is not sufficient. Consider the function $f: \{1,2,3\} \rightarrow \{a, b, c\}$ where all elements of the domain map to 'a'.

9. Recall that a characteristic function can be written as a vector V of 1's and 0's, and can be used to describe the elements of a subset of a set S. If S has size n, then V will be a vector of length n.

Suppose that $S = \{a,b,c\}$

What is the vector notation for describing the subset $\{a,b\}$?

$\{1,1,0\}$