CSE 202 Calibration Homework
Spring, 2008
All parts are worth 20 points. Due April 10

Recurrence Let $T(n)$ be the function given by the recursion: $T(n) = nT(\lfloor\sqrt{n}\rfloor)$ for $n > 1$ and $T(1) = 1$. Is $T(n) \in O(n^k)$ for some constant k, i.e. is T bounded by a polynomial in n? Prove your answer either way. (Note: logic and definition of O notation are more important than exact calculations for this problem.)

Reasoning about order Let $f(n)$ be a positive, integer-valued function on the natural numbers that is non-decreasing. Show that if $f(2n) \in O(f(n))$, then $f(n) \in O(n^k)$ for some constant k. Is the converse also always true?

Triangles: A triangle in a graph are 3 nodes any two of which are adjacent. Present two algorithms for determining whether a graph has a triangle, one if the graph is given as an adjacency matrix and the other if it is given in adjacency list format. Analyze these algorithms in terms of both the number of nodes n and the number of edges m.

Base Conversion Present and analyze an $O(n^2)$ time algorithm that inputs an array of n base 10 digits representing a positive integer in base 10 and outputs an array of base 2 bits representing the same integer in base 2. Count each operation on a single digit as a step, e.g., adding two n bit binary strings takes time $O(n)$ since one addition involves $O(n)$ bit operations. Be sure to prove that your algorithm is correct.

Implementing Base Conversion . Implement the above algorithm, and test it on many random n bit strings for $n = 128$, $n = 256$, $n = 512$, $n = 1024$, $n = 2048$, $n = 4096$, $n = 8192$, $n = 16384$, and $n = 32768$. Plot time vs. input size on a log vs. log curve. Does the algorithm's observed time fit the analysis? Why or why not?