CSE 200
Computability and Complexity
Homework 3
NP, Completeness, and Reductions
Part 1
Due Wed. May 28

May 5, 2008

Give proofs for each problem. Proofs can be high-level, but be precise. You may use without giving a proof any result proved in class or in the textbook. In particular, to prove NP-completeness, it suffices to give a reduction from any of the NP-complete problems from the text or from class. However, you must show your reduction is valid, by showing the equivalence of the constructed instance and the original.

SAT restriction Prove that the 3SAT problem remains NP-complete when restricted to formulas where each variable appears in at most 3 clauses. Remember that the input to 3SAT is a CNF formula with *at most* three variables per clause, so clauses of size 2 and 1 are also permissible.

neq SAT Problem 7.24 from the Sipser text, page 296.

Max-Cut Problem 7.25 from the Sipser text, page 296.

Closure under reductions Prove that NP is closed under polynomial-time Turing reductions if and only if $NP = Co - NP$.

Sodoku The *sudoku* problem of size n is as follows. The input is an $n^2 \times n^2$ matrix M whose entries are either "blank" or an integer between 1 and n^2. A solution fills in the blank spaces with integers between 1 and n^2. The following constraints must be met: Each integer from 1 to n^2 appears exactly once in each row, in each column, and in each $n \times n$ sub-matrix of the form $M[jn+1...(j+1)n][in+1..(i+1)n]$ for each $0 \leq i, j \leq n-1$. The problem is to find any solution meeting the constraints, or return "no solution possible" if there is no such solution.

Give at least two ways of reducing sudoku to $CNFSAT$. For each, specify how many variables and clauses are created as a function of n, and also the size of the different categories of clauses (e.g., clauses of n variables vs. clauses with n^2 variables.) Give a guess for which reduction methods you would expect to work well with SAT solvers and why (see next assignment).