

UCSD CSE 101 MIDTERM 1, Winter 2008

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Name: _____

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Read all of the following information before starting the exam.

- This test has 3 problems totaling 50 points.
- You have 50 minutes to work on the exam. If you finish early, please be quiet when you leave. *Out of courtesy to those still working on the exam, no one will be allowed to turn in their exam during the last 10 minutes.*
- Show all work, clearly and in order, if you want to get full credit. You may lose points if you do not show how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Be clear and to the point. You will lose points for rambling and for **incorrect or irrelevant** statements.
- Make sure you have all 6 pages. Use the scratch page and/or the reverse sides of pages as necessary - and ask if you need extra sheets of paper. Good luck!

1. (12 points) For (a) and b), indicate whether $f(n)$ is $O(g(n))$, $\Omega(g(n))$ or $\Theta(g(n))$, and justify briefly. Your answer must be as specific as possible.

a. (3 pts) $f(n) = \log 2n$ and $g(n) = \log 3n$

b. (3 pts) $f(n) = n^{1/2}$ and $g(n) = n^{2/3}$

c. (6 pts) Give the running time of the following algorithm. Your answer should be of the form $O(f(|V|, |E|))$, for some specific choice of f . Explain your answer briefly.

FindMaxDegree(G: an undirected graph given in adjacency list form):

1. $max = 0$
2. For every vertex $v \in V$
 - (a) Set $count = \text{number of edges in G of the form } (v, w), w \in V$.
 - (b) If $count > max$, then $max = count$
3. return max

2. (20 points) Definition: A *bipartite graph* is a graph $G = (V, E)$ whose vertices can be partitioned into two sets ($V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$) such that there are no edges between vertices belonging to the same set.

(For instance, if two vertices u, v are elements of V_1 , then there is no edge between u and v . A bipartite graph is also called a *2-colorable* graph.)

a. (8 pts) Prove that an undirected graph G is bipartite if and only if it contains no cycles of odd length.

($\rightarrow$) (If G is bipartite, it has no cycles of odd length.)

($\leftarrow$) (If G has no cycles of odd length, it is bipartite.)

b. (12 pts) Give a linear-time algorithm to determine whether an undirected graph is bipartite. Analyze its running time and prove that it is correct.

3. (*18 points*) Given an undirected graph $G = (V, E)$ with positive edge weights, along with a particular node $v_0 \in V$. Give an efficient algorithm (*6 pts*) for finding the shortest path distances between all pairs of nodes, with the restriction that all shortest paths must pass through v_0 . Analyze the running time of your algorithm (*6 pts*) and prove its correctness (*6 pts*).

Scratch Page

(please do not remove this page from the test packet)